Hierarchical majority-rule sorting models for temporal multi-criteria decision aiding

Arthur Valko, Patrick Meyer, Alexandru-Liviu Olteanu

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Arthur Valko[1,2], Patrick Meyer[1], Alexandru-Liviu Olteanu[3]

[1] Chair of Cyber defence of naval systems
arthur.valko@ecole-navale.fr
[2] IMT Atlantique, Lab-STICC, Univ. Bretagne Loire, F-29238 Brest, France
{arthur.valko,patrick.meyer}@imt-atlantique.fr
[3] Lab-STICC, Université Bretagne Sud, Lorient, France
alexandru.olteanu@univ-ubs.fr

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1 Introduction

In certain applications, evaluations of decision alternatives on multiple criteria may vary in order to reflect its performance, for instance, at various time steps in the past and/or in the future. Taking all of these factors into account at once may render the problem too difficult from the perspective of the decision-maker (DM). For this reason, we propose to divide the decision problem into a hierarchy of sub-problems. We also consider the case where a majority-rule sorting model, or MR-Sort, has been selected to model the preferences of the DM.

The motivation behind this work is mainly based on a practical context linked to reacting to cyber-attacks of naval systems. Following a cyber-attack, a DM (the ship captain, for example) may wish to take an action in order to restore some or all of the ship's functions. The time when these functions will be restored may also play a role based on the mission requirements at each time step. To our knowledge, multi-criteria decision aiding (MCDA) has only been poorly applied to the cyber-defence context (see, e.g. [2]).

2 Multi-criteria Decision Aiding and time management

MCDA focuses on the problems, methods and tools which may be used in order to assist a DM in reaching a decision when faced with a set of alternatives, characterized by multiple, often conflicting, criteria [4]. Three types of decision problems may be found (choosing the best option, sorting into predefined ordered categories or ranking alternatives from best to worst). Various models have been proposed, originating from two methodological schools : *outranking* methodologies, where any two alternatives are compared pair-wisely on the basis of their evaluations on the set of criteria ; *multi-attribute value theory*, where we construct a numerical representation of the decision maker's preferences on the set of alternatives [1].

MCDA methodologies are mainly built to work with a single evaluation of each alternative for each criterion. This means that an alternative is represented through a vector of evaluations. However, as stated by our application context, in certain cases, each criterion may become a time series, therefore our alternatives are represented by a matrix of evaluations, in which, each line corresponds to a criterion, whereas each column corresponds to a time step.

To manage this new representations of an alternative, we have two options : first, one could consider each evaluation of a criterion on each time step as a new criterion (and thus return to a "vector" representation) ; second, one could try to aggregate the evaluations of a criterion for all time steps into a single evaluation [5] (for example by giving more or less importance to various time steps). The first option might lead, for example during the preference elicitation phase, the DM to consider a huge number of criteria all at once. With the second option, one might miss a large amount of information which was present in the original time series.

3 Handling time-related impacts through hierarchical sorting models

We propose a two-level decomposition of the considered problem by splitting it into sub-problems, one for each time step. The purpose of this is to simplify the decision by considering each time step independently of the rest , and then aggregating these results at a global scale.

First the problem is divided into sub-problems across the time dimension and then each of them is modeled independently using an MR-Sort model. A last MR-Sort model is needed to combine the results of the underlying models into a global one. When assigning an alternative to a category, its matrix of evaluations is thus split along its columns, the vector of evaluations corresponding to each column being fed into the MR-Sort model of the corresponding time step. The assignments of each of these models correspond to the perception of the DM with respect to the overall performance of the alternative on each time step. They form a new vector of evaluations that will be fed into the global MR-Sort model, which in turn provides the final assignment.

The main contribution of this work is the proposal of this hierarchical MR-Sort model and an exact approach, based on Mixed Integer Programming, for inferring the its parameters. Currently such inference approaches only exist for independent MR-Sort models [3]. The proposed approach infers the parameters of all of the sub-models at the same time, taking as input holistic judgments of the DM on the overall quality of a decision alternative on multiple criteria and across multiple time-steps. We also study the efficiency of the inference algorithm, as well as the expressiveness and generalization power of the hierarchical model through numerical experiments on artificially generated datasets. The first results show that the hierarchical model better generalizes on test data, but uses, as expected, more time for the learning phase.

Références

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