EMMI Rapid Reaction Task Force
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Microscopic Models of Energy Loss and transport coefficients:
The Nantes Approach

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Motivation and context

- Most of the *interesting* HF observables so far: located at *intermediate* $p_T$
  ($\approx 3$ GeV-50 GeV)

- Intermediate $p_T$: hope that pQCD (or pQCD inspired models) apply (as compared to low $p_T$)

- Intermediate $p_T$: mass effect still present and thus hope to learn something more as compared to large $p_T$

Low (Energy conservation under control)

- Braaten-Thoma + Gunion- Bertsch
  $\approx$ Bethe-Bloch + Bethe-Heitler

Intermediate

Coherence effects

High (coherence under control)

- Finite E + finite mass corrections

BDMPS-Z, DGLV, ASW, …
  $\approx$ LPM

Approach pursued in our **models**… Unfortunately too many of them

=>$\text{Need for falsification (more observables; IQCD): Azimuthal correlations?}$
Starting from Combridge (79) as a basis:

$$\sum |M|^2 = \frac{64 \pi^2 \alpha^2(q^2)}{q} \frac{(M^2 - u)^2 + (s - M^2)^2 + 2 M^2 t}{t^2}$$

$$\sum |M|^2 = \pi^2 \alpha^2(q^2) \left[ \frac{32 (s - M^2)(M^2 - u)}{t^2} + \frac{64 (s - M^2)(M^2 - u) + 2 M^2 (s - M^2)}{q} \frac{(s - M^2)^2}{(s - M^2)^2} \right]$$

$$\quad + \frac{64}{q} \frac{(s - M^2)(M^2 - u) + 2 M^2 (M^2 - u)}{(M^2 - u)^2} + \frac{16}{q} \frac{M^2 (4 M^2 - t)}{(s - M^2)(M^2 - u)}$$

$$\quad + \frac{16}{q} \frac{(s - M^2)(M^2 - u) + M^2 (s - u)}{t (s - M^2)} - \frac{16}{q} \frac{(s - M^2)(M^2 - u) - M^2 (s - u)}{t (M^2 - u)} \right]$$

However, t-channel is IR divergent => model $S$
Naïve regulating of IR divergence:

\[
\frac{1}{t} \quad \rightarrow \quad \frac{1}{t - \mu^2}
\]

With \(\mu(T)\) or \(\mu(t)\)

Models A/B: 2 customary choices

\[
\mu^2(T) = m_D^2 = 4\pi \alpha_s(1+3/6)\lambda T^2
\]

\[
\alpha_s(Q^2) \rightarrow \begin{cases} 0.3 \text{ (mod A)} \\ \alpha_s(2\pi T) \text{ (mod B)} \approx 0.3 \end{cases}
\]

(Svetitsky: 0.5; equil time of 1fm/c) !!!

\[
\begin{array}{|c|c|c|}
\hline
\text{T (MeV)} & \text{p (GeV/c)} & 10 & 20 \\
\hline
200 & 0.18 & 0.27 \\
400 & 0.35 & 0.54 \\
\hline
\end{array}
\]

\[ dx \]

... of the order of a few %!

OBE model, NOT pQCD at finite T !!!
Educated: Calibrating on HTL…

permits to fix the effective mass $\mu$
Heavy fermion of mass $M$ probes the medium via virtual fermion of momentum $q$.

**Region I:** $q > q^*$: hard; close collisions; individual; incoherent.

**Region II:** $q < q^*$: soft; far collisions; collective; coherent; macroscopic.

Relying on the smallness of the coupling constant $\frac{T e T}{r D} \approx \frac{1}{e^2 T}$, we have:

$$\frac{1}{M} \ll r = \frac{1}{T} \ll \frac{1}{q^*} \ll r_D \approx \frac{1}{e T} \ll \lambda \approx \frac{1}{e^2 T}$$

Debye radius $r_D$.
Braaten-Thoma:
(Peshier – Peigné)

Low $|t|$: large distances

HTL: collective modes

$$G_{\mu\nu}(Q) = \frac{-\delta_{\mu0}\delta_{\nu0}}{q^2 + \Pi_{00}} + \frac{\delta_{ij} - \hat{q}_i \hat{q}_j}{q^2 - \omega^2 + \Pi_T}$$

$$\frac{dE_{soft}}{dx} = \frac{2}{3} \alpha m_D^2 \ln \left( \frac{\sqrt{t^*}}{m_D / \sqrt{3}} \right) + \ldots$$

SUM: $$\frac{dE}{dx} = \frac{2}{3} \alpha m_D^2 \ln \left( \frac{\sqrt{ET}}{m_D / \sqrt{3}} \right)$$

HTL: convergent kinetic
(matching 2 regions)

$$|t^*|$$

Large $|t|$: close coll.

$$G_{\mu\nu}(Q) = \frac{-\delta_{\mu\nu}}{q^2 - \omega^2}$$

$$\frac{dE_{hard}}{dx} = \frac{2}{3} \alpha m_D^2 \ln \left( \frac{\sqrt{ET}}{\sqrt{t^*}} \right) + \ldots$$

Indep. of $|t^*|$!

(provided $g^2 T^2 \ll |t^*| \ll T^2$)
In QGP: $g^2 T^2 > T^2$ !!!

**BT: Not Indep. of $|t^*|$!**

Our solution: Introduce a semi-hard propagator -- $1/(t-v^2)$ -- for $|t| > |t^*|$ to attenuate the discontinuities at $t^*$ in BT approach.

**Prescription:** $v^2$ in the semi-hard prop. is chosen such that the resulting E loss is maximally $|t^*|$-independent.

This allows a matching at a natural value of $|t^*| \approx T$... Not an increase wrt Braaten-Thoma.
Model C: optimal $\mu^2$

**THEN**: Optimal choice of $\mu$ in our OBE model:

\[
\frac{\alpha_s(2\pi T)}{t - \mu^2} \mu^2(T) = \kappa m_D^2(T)
\]

With $\kappa \approx 0.15$

with $m_D^2 = 4\pi\alpha_s(2\pi T)(1+3/6)xT^2$

\[
dE_{coll}(c) \frac{dx}{dx} \text{ ... factor 2 increase w.r.t. mod B (not enough to explain } R_{AA}\text{)}
\]

Convergence with “pQCD” at high T

<table>
<thead>
<tr>
<th>$T$(MeV) \ $p$(GeV/c)</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>0.36 (0.18)</td>
<td>0.49 (0.27)</td>
</tr>
<tr>
<td>400</td>
<td>0.70 (0.35)</td>
<td>0.98 (0.54)</td>
</tr>
</tbody>
</table>
Refined: *running coupling constant*

Motivation: Even a fast parton with the largest momentum $P$ will undergo collisions with moderate $q$ exchange and large $\alpha_s(Q^2)$. The running aspect of the coupling constant has been “forgotten/neglected” in most of approaches.

Open question: long range behaviour and renormalisation at finite temperature
A Peshier: $\alpha_S$ not fixed at the right scale

Running of $\alpha_S$ (Peshier 06) in collisional E loss

Usually

$$\frac{dE_j}{dx} = \sum_s \int_{k^3} \rho_s(k) \Phi \int dt \frac{d\sigma_{js}}{dt} \omega$$

with

$$\Phi \int_{t_1}^{t_2} dt \frac{d\sigma_{js}}{dt} \omega = \pi C_{js} \alpha^2 \int_{t_1}^{t_2} \frac{dt}{t} = \frac{\pi C_{js} \alpha^2}{k} \ln \frac{t_1}{t_2}$$

Doing it more cautiously

$$\Phi \int_{t_1}^{t_2} dt \frac{d\sigma_{js}}{dt} \omega = \frac{\pi C_{js}}{k b_0^2} \int_{t_1}^{t_2} \frac{dt}{t \ln^2(|t|/\Lambda^2)} = \frac{\pi C_{js}}{k b_0^2} \ln \left(\frac{|t_2|}{\Lambda^2}\right) - \frac{\pi C_{js}}{k b_0^2} \ln \left(\frac{|t_1|}{\Lambda^2}\right)$$

Dominated by the soft scale

No log(E) increase. UV conv. for $t_1 \to \infty$

Softer scale $\Rightarrow$ larger E loss !!!

"In fact, $\sigma$ with running coupling … an order of magnitude larger than expected from the widely used expression $\sigma_{\alpha \text{fix}} \propto \alpha^2(Q^2T)/\mu^2$. Thus, the present approach gives a consistent and simple explanation of phenomenologically inferred large cross sections found in transport models."
IR safe. The detailed form very close to $Q^2 = 0$ is not important does not contribute to the energy loss. Large values for intermediate momentum-transfer => larger cross section.

Of course, still a lot of uncertainties in the choice of this essential quantity !!!
μ-local-model: medium effects at finite T in t-channel

Large $|t|$:

- $\alpha_{\text{eff}}(Q^2, T=0)$
- $|t^*|$ (Max. insensitivity)

Low $|t|$:

- HTL: collective modes

Semi-hard

$$\frac{\alpha_{\text{eff}}(t)}{t - \lambda m_D^2(T, t)}$$

$\lambda = 0.11$

OGE with effective polarisation

$$\mu^2(T) = 0.2 \ m_{\text{Dself}}^2(T)$$

$$m_{\text{Dself}}^2(T) = (1+n_f/6) \ 4\pi\alpha_{\text{eff}}(m_{\text{Dself}}^2) \ T^2$$

Bona Fide running HTL:

$\alpha_s \rightarrow \alpha_s(t)$ in $\Pi_L$ and $\Pi_T$
Drag coefficient $A$ ($\langle d\langle p\rangle/dt \rangle$)

At large $p$: moderate mass dependence

<table>
<thead>
<tr>
<th>$\alpha_s$</th>
<th>$\mu^2$</th>
<th>Line form</th>
<th>Line color</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.3</td>
<td>$m_D^2$</td>
<td>Dotted thin</td>
</tr>
<tr>
<td>B</td>
<td>$\alpha_s(2\pi T)$</td>
<td>$m_D^2$</td>
<td>Dashed thin</td>
</tr>
<tr>
<td>C</td>
<td>$\alpha_s(2\pi T)$</td>
<td>0.15$m_D^2$</td>
<td>Full thin</td>
</tr>
<tr>
<td>D</td>
<td>Running, Eq. (17)</td>
<td>$\tilde{m}_D^2$</td>
<td>Dashed bold</td>
</tr>
<tr>
<td>E</td>
<td>Running, Eq. (17)</td>
<td>0.2$\tilde{m}_D^2$</td>
<td>Full bold</td>
</tr>
<tr>
<td>F</td>
<td>Running, Eq. (17)</td>
<td>$0.11(6\pi \alpha_{\text{eff}}(1) T^2)$</td>
<td>Dashed dotted bold</td>
</tr>
</tbody>
</table>
μ-local-model: Eff. Running $\alpha_s$ vs lQCD

$T=0$

$\alpha_{qq}(r) \equiv \frac{3}{4} r^2 \frac{dV(r)}{dr}$

O. Kaczmarek & F. Zantow (KZ) (n$_f$=2 QCD), P.R.D71 (2005) 114510

Genuine non-pert (string)

optimal $\mu$, running $\alpha_{\text{eff}}$

$V$:$\omega=0$ sector; $dE/dx$: finite $\omega$

Finite $T$

T$\approx$1.1 $T_c$

$\frac{dV}{dr}$ [GeV/fm]

T$\approx$1.5 $T_c$

$\frac{dV}{dr}$ [GeV/fm]

Merging at $\approx$2 $T_c$

Some overshooting at large distance

KZ P.R. D71 (2005)  

Differential cross sections

**Qq → Qq**

- \( \frac{d\sigma_{q\bar{q}\rightarrow q\bar{q}}}{dt} \) (a.u.)
- Large enhancement of both cross sections at small and intermediate \(|t|\)
- Little change at large \(|t|\)

\( \alpha(2\pi T), \mu = m_D \)

\( \mu^2(t) = 0.11 + 6\pi \alpha(t) T^2 \)

**Qg → Qg**

- \( \frac{d\sigma_{g\bar{g}\rightarrow g\bar{g}}}{dt} \) (a.u.)
- "standard"

\( \alpha(2\pi T), \mu = m_D \)

\( \mu^2(t) = 0.11 + 6\pi \alpha(t) T^2 \)

\( \mu^2 = m_D^2 \quad \text{\&} \quad \mu^2 = m_D^2 \text{self}(T) \)
\( \mu \)-local-model: Eff. Running vs fixed \( \alpha_s \)

Good agreement with PP for large \( T \) and large \( P \)

Running \( \alpha_s \) is more than a cranking of BT (different shapes and \( T \)-dependences)

Conclusions:
Running $\alpha_s$ : some Energy-Loss values

\[ \frac{dE_{\text{coll}}(c/b)}{dx} \]

<table>
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<tbody>
<tr>
<td>200</td>
<td>1 / 0.65</td>
<td>1.2 / 0.9</td>
</tr>
<tr>
<td>400</td>
<td>2.1 / 1.4</td>
<td>2.4 / 2</td>
</tr>
</tbody>
</table>

\[ \approx 10\% \text{ of HQ energy} \]

Drag coefficient

E: optimal $\mu$, running $\alpha_{\text{eff}}$
C: optimal $\mu$, $\alpha_s(2\pi T)$

Transp. Coef ...

... of expected magnitude to reproduce the data (we “explain” the transp. Coeff. in a rather parameter free approach).
Several issues

1. Non perturbative aspects (beyond Born). Usually in convergent kinetic:

\[ \text{RPA} + \prod \cdot \cdot \cdot \]

Ladders necessary at short distance (large force)
Several issues

2. $\lambda$ at small momentum?

Reduction of the interaction range

$\lambda = 0.11$

$T = 0.2, 0.3 \& 0.4 \text{ GeV} \quad N_f = 3 \quad M = 1.5 \& 5 \text{ GeV}$

$\nu^2/m_D^{2\text{eff}}$ vs $\frac{dP}{dx}$

$A(\text{GeV/fm})$ vs $p(\text{GeV/c})$

c–quarks

$T = 0.4 \text{ GeV}$

$\lambda = 0.11$

fixed at $\alpha_s(2\pi T)$

$\lambda_{\text{opt}}$

b–quarks

$T = 0.4 \text{ GeV}$

$\lambda = 0.11$

fixed at $\alpha_s(2\pi T)$

$\lambda_{\text{opt}}$
Several issues

3. How to deal with the genuinely NP part?
Transport coefficients (1)

Only the elastic contribution
Gathering all rescaled models (coll. and radiative) compatible with RHIC $R_{AA}$:

The drag coefficient reflects the average momentum loss (per unit time) => large weight on $x \sim 1$

For too large $p_T$, $L^2$ terms dominate => transport coefficients are not the relevant objects
Gathering all rescaled models (various prescriptions for $\mu$ and $\alpha_s$):

AdS/CFT too large to reproduce experimental data?! Against the conclusion of Akamatsu et al (?)

(E-loss plays a dominant role, but not the only parameter)
comparer avec hirano et al qui parviendraient à reproduire le RAA
The Monte Carlo @ Heavy Quark Generator

No force on HQ before thermalization of QGP (0.6 fm/c)

Evolution according to Bjorken time

Preequilibrium

Quarkonia formation in QGP through c+c→Ψ+g fusion process

(hard) production of heavy quarks in initial NN collisions (NLO or FONLL or any pp generator + k_T broad. (0.2 GeV^2/coll)

\[ \frac{1}{2\pi} \frac{d\sigma}{dp_T} (\text{mb/GeV}^2) \]

HQ Lectures Nantes
The Monte Carlo @ Heavy Quark Generator

Bulk Evolution: non-viscous hydro (Heinz & Kolb) → T(M) & v(M)

Evolution of HQ in bulk: Fokker-Planck or reaction rate + Boltzmann (no hadronic phase)

Recently: coupling to EPOS2 (3) instead of KH

HQ Lectures Nantes
D/B formation at the boundary of QGP (or MP) through coalescence of c/b and light quark (low $p_T$) or fragmentation (high $p_T$)

Bulk Evolution: non-viscous hydro (Heinz & Kolb) $\rightarrow T(M)$ & $v(M)$

Evolution of HQ in bulk: Fokker-Planck or reaction rate + Boltzmann (no hadronic phase)

Nothing spectacular at freeze-out (quarkonia are white objects already)
Boltzmann vs Langevin

Deviation from Einstein relation with native coefficients

\[ f_{\text{asympt}}(\varphi) \]

\[ f_{\text{asympt}}(\varphi)/e^{-\varphi} \]

\[ |M_{\text{grazing}}(s, t, \mu; \nu)|^2 = \frac{(s - m^2)^2 \mu^{4(\nu-1)}}{(\mu^2 - t)^{2\nu}} \]

\[ \varphi = \frac{E - 8m_c}{T} \]
Boltzmann vs Langevin

2 corrections prescriptions:

- **VHR:** \( B_L^{\text{therm VHR}}(p) = E_p T A / p \)

- **Gossiaux (historical)** 
  \[
  \frac{B_L^{\text{therm}}(p)}{B_T^{\text{therm}}(p)} = \left( \frac{B_L(p)}{B_T(p)} \right)^\beta
  \]
  \( \beta = 0.25 \)

![Graph showing the comparison between different models]
Boltzmann vs Langevin

\[ \langle p_z \rangle \text{(GeV/c)} \]

- **Boltzmann**
- **FP native**
- **FP therm Gossiaux**
- **FP therm VH&H**

\( T = 300 \text{MeV} \)
Boltzmann vs Langevin

\[ \sigma(p_z)(\text{GeV/c}) \]

- Boltzmann
- FP
- FP therm
- FP therm VH&R

\[ p_{\text{in}} = 20\text{GeV/c} \]

\[ T = 300\text{MeV} \]

\[ \sigma(p_{\perp})(\text{GeV/c}) \]

- Boltzmann
- FP
- FP therm
- FP therm VH&R

\[ p_{\text{in}} = 20\text{GeV/c} \]

\[ T = 300\text{MeV} \]

\[ \sigma(p_T)(\text{GeV/c}) \]

- Boltzmann
- FP
- FP therm
- FP therm VH&R

\[ \text{pp-LHC} \]

\[ \text{pp-RHIC} \]

\[ T = 400\text{MeV} \]

\[ T = 300\text{MeV} \]

Just drag
Boltzmann vs Langevin

Evolution in a finite T stationary medium (infinite)

Both tuned FP ok,… Native FP has less RAA (more longitudinal fluctuations dating from Einstein violation)
Induced Energy Loss

Generalized Gunion-Bertsch (NO COHERENCE) for finite HQ mass, dynamical light partons

Eikonal limit (large \( E \), moderate \( q \))

\[
\omega \frac{d^3 \sigma_{\text{rad}}^{x \ll 1}}{d\omega d^2 k_\perp dq_\perp} = \frac{N_c \alpha_s}{\pi^2} (1 - x) \times \frac{J_{\text{QCD}}^2}{\omega^2} \times \frac{d\sigma_{\text{el}}^{Qq}}{dq_\perp^2}
\]

with

\[
\frac{J_{\text{QCD}}^2}{\omega^2} = \left( \frac{-k_\perp}{k_\perp^2 + x^2 M^2 + (1 - x)m_g^2} - \frac{\tilde{k}_\perp - \tilde{q}_\perp}{(\tilde{k}_\perp - \tilde{q}_\perp)^2 + x^2 M^2 + (1 - x)m_g^2} \right)^2
\]

Gluon thermal mass \( \sim 2T \) (phenomenological; not in BDMPS)

Quark mass

Both cures the collinear divergences and influence the radiation spectra (dead cone effect)

Dominates as small \( x \) as one "just" has to scatter off the virtual gluon \( k' \)
Incoherent Induced Energy Loss

… & finite energy!

Finite energy lead to strong reduction of the radiative energy loss at intermediate $p_T$

Formation time for a single coll.

At 0 deflection:

\[ t_f \approx \frac{2(1-x)\omega}{(k_\perp - q_\perp)^2 + x^2 M^2 + (1-x)m_g^2} \]

For \( x > x_{cr} = m_g / M \), gluons radiated from heavy quarks are resolved in less time than those light quarks and gluon radiation process less affected by coherence effects in multiple scattering.

For \( x < x_{cr} = m_g / M \), basically no mass effect in gluon radiation.

Dominant region for quenching

Dominant region for average E loss

\[ l_{f,\text{sing}} \approx \frac{2x(1-x)E}{m_g^2 + x^2 M^2} \]
A first criteria

Comparing the formation time (on a single scatterer) with the mean free path:

\[ \lambda(T) \]

Coherence effect for HQ gluon radiation:

\[ \frac{E}{M} \gtrsim m_g \lambda_Q \sim \frac{1}{g_s} \]

Maybe not completely foolish to neglect coherence effect in a first round for HQ.

(of course depends on the physics behind \( \lambda_Q \))

Mostly coherent

Mostly uncoherent

RHIC

LHC

(will provide at least a maximal value for the quenching)
Our basic ingredients for HQ energy loss

Coherent Induced Radiative

Formation time picture: for $l_{f,mult} > \lambda$, gluon is radiated coherently on a distance $l_{f,mult}$

Model: all $N_{coh}$ scatterers act as a single effective one with probability $p_{Ncoh}(Q_{\perp})$ obtained by convoluting individual probability of kicks

$$\frac{d^2 I_{eff}}{dz d\omega} \sim \frac{\alpha_s}{N_{coh} \bar{\lambda}} \ln \left( 1 + \frac{N_{coh} \mu^2}{3 (m_g^2 + x^2 M^2 + \sqrt{\omega q})} \right)$$

[arXiv:1209.0844] (Hard Probes 2012)
Monte Carlo Implementation (rad)

I) For each collision with a given $q_\perp$, we define the conditional probability of radiation:

$$r(q_\perp) := \int_0^\infty \frac{d^2\sigma_{\text{rad}}}{d\omega dq_\perp^2} d\omega$$

In practice, $\omega_{\text{min}} = 5\%$ E to avoid IR catastrophe

II) For each collision with a given invariant mass squared $s$, we define the conditional total probability of radiation:

$$\tilde{r}(s) = \frac{\sigma_{\text{rad}}}{\sigma_{\text{el}}} \approx \frac{\int_{-|t|_{\text{max}}}^0 r(\sqrt{-t}) \frac{d\sigma_{\text{el}}^Q q(t)}{dt} dt}{\int_{-|t|_{\text{max}}}^0 \frac{d\sigma_{\text{el}}^Q q(t)}{dt} dt}$$

Probes the elastic cross section at larger values of $t$ => less sensitive to $\alpha_{\text{eff}}$ at small $t$-values

Threshold for radiation

HQ Lectures Nantes
III) For a given HQ energy $E$, we sample the entrance channel according to the thermal distribution of light quarks and gluons and $\sigma_{el}(s)$ and accept according to the conditional probability $\tilde{r}(s)$.

IV) We sample “downwards” $q_\perp$, $\omega$ and then $k_\perp$.

Hard shocks with $|t|>25\%$ s are rejected (not treated properly in our formalism).

V) $P^+ \rightarrow (1-x) P^+$ and transverse kick of $q_\perp - k_\perp$.

Fixed $\alpha_s$ Approximation:

In “reality”, several collisions at intermediate $t$-values accumulate.

VI) Reject if out of phase-space.