Learning Chaotic and Stochastic Dynamics from Noisy and Partial Observation using Variational Deep Learning

Duong Nguyen, Said Ouala, Lucas Drumetz, Ronan Fablet

LEARNING CHAOTIC AND STOCHASTIC DYNAMICS FROM NOISY AND PARTIAL OBSERVATION USING VARIATIONAL DEEP LEARNING

Duong Nguyen[1], Said Ouala[1], Lucas Drumetz[1] and Ronan Fablet[1]

I. CONTEXT AND PROPOSED APPROACH

Although many works have recently successfully provided proofs of concept for data driven approaches of learning dynamical systems under ideal conditions, i.e. noise-free and high sampling frequency, dealing with real life data where the measurements are usually noisy and can be partially and irregularly sampled remains challenging.

We propose a new method, called DAODEN (Data-Assimilation-based Ordinary Differential Equation Network), which can explicitly capture the stochastic components of both the process of interest and the observation system. It involves two main modules: an inference module which aims to reconstruct the hidden states of the systems from damaged observations, and a generative model which addresses the dynamics of the system and the observation model. The proposed learning strategy relies on a variational learning setting. By construction, it guarantees performance similar to state-of-the-art schemes under ideal experimental settings.

Specifically, given a series of observations $\mathbf{x}_{1:T}$, which can be noisy, partially and irregularly sampled of a dynamical system, DAODEN supposes that the generation process of $\mathbf{x}_{1:T}$ relies on a series of true states $\mathbf{z}_{1:T}$. The inference module in DAODEN is an LSTM-based network that reconstructs $\mathbf{z}_t$ from $\mathbf{x}_{1:T}$: $q_\phi(\mathbf{z}_t|\mathbf{x}_{1:T})$. The generative module involves the parametrization of the dynamics of the hidden states $p_\theta(\mathbf{z}_{t+1}|\mathbf{z}_t)$, modeled by a neural network; and an observation distribution $p(\mathbf{x}_k|\mathbf{z}_k)$, usually known. DAODEN maximizes the Evidence Lower BOund (ELBO) of the log likelihood $\ln p(\mathbf{x}_{1:T})$ over $\{\theta, \phi\}$. This optimization results in the surrogate model of the dynamics of the considered system.

II. EXPERIMENT AND RESULT

As illustration of the proposed framework, we first consider an application to the identification of an ODE representation given noisy and irregularly sampled observations, here for Lorenz-63 system. Using a Bilinear Neural Network (BiNN) [1] for the dynamical module, we show that DAODEN significantly outperforms the direct learning of the BiNN model from the observation data both in the short-term prediction error and the long-term topology.

Compared with previous works, DAODEN can also capture the stochasticity of dynamical systems, such as in the Lorenz 63 stochastic system [2], as presented in Fig. 1.

TABLE I: Short term prediction error ($e4$) and the first Lyapunov exponent (λ_1) of models trained on noisy and partially observed Lorenz63 data with a missing rate of 80%. The results are averaged over 50 test sequences.

Model		std_{noise}/std_{signal} 8.5 %	33.3%
BiNN	$e4$	0.348±0.327	0.372±0.238
	λ_1	1.116±0.026	0.329±0.033
DAODEN	$e4$	0.089±0.062	0.162±0.104
	λ_1	0.892±0.011	0.859±0.013

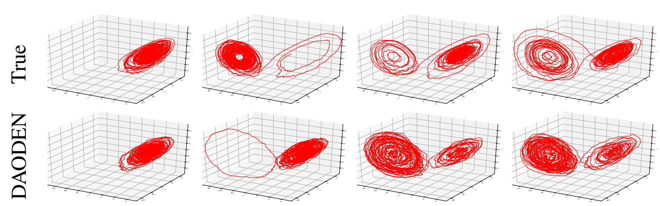

Fig. 1: Attractors generated by the true Lorenz63 stochastic system (top), and by DAODEN (bottom). The true Lorenz63 stochastic system and DAODEN system are stochastic, hence each runtime we obtain a different sequence, even with the same initial condition. The model was trained on noisy observations, with a noise level of 33.3%.

[1]IMT Atlantique, Lab-STICC, 29238 Brest, France

REFERENCES

[1] R. Fablet, S. Ouala, and C. Herzet, "Bilinear Residual Neural Network for the Identification and Forecasting of Geophysical Dynamics," in *2018 26th European Signal Processing Conference (EUSIPCO)*, pp. 1477–1481, Sept. 2018. ISSN: 2219-5491.

[2] B. Chapron, P. Dérian, E. Mémin, and V. Resseguier, "Large-scale flows under location uncertainty: a consistent stochastic framework," *Quarterly Journal of the Royal Meteorological Society*, vol. 144, no. 710, pp. 251–260, 2018.