Hub-and-spoke network design under congestion: A learning based metaheuristic

Maryam Karimi-Mamaghan, Mehrdad Mohammadi, Amir Pirayesh, Amir Mohammad Karimi-Mamaghan, Hassan Irani

Hub-and-Spoke Network Design under Congestion: A Learning based Metaheuristic

Maryam Karimi-Mamaghan [a*], Mehrdad Mohammadi [a*], Amir Pirayesh [b], Amir Mohammad Karimi-Mamaghan [c], Hassan Irani [d]

[a] *IMT Atlantique, Lab-STICC, UBL, F-29238 Brest, France*
[b] *Centre of Excellence in Supply Chain and Transportation (CESIT), KEDGE Business School, Bordeaux, France*
[c] *Department of Electrical and Computer Engineering, University of Tehran, Tehran, Iran*
[d] *Construction Management and Engineering ,Civil and Architecture Faculty, Islamic Azad university, Science and Research Branch, Tehran, Iran*

Abstract

This paper models a single allocation multi-commodity hub-and-spoke network problem through a bi-objective mathematical model, considering the congestion in both hubs and connection links. A novel aggregation model is developed based on a general GI/G/c queuing system to evaluate the congestion of the flow of the multiple products in the hubs. The proposed model is then solved using a novel learning-based metaheuristic based on NSGA-II, *k*-Means clustering method, and an Iterated Local Search (ILS) algorithm. The proposed model and solution algorithm are validated through a set of experiments against optimal solutions, and benchmarked against four existing well-known algorithms.

Keywords: Hub-and-spoke network design; Bi-objective optimization; Congestion; Queuing network; Machine Learning; *k*-Means clustering method; Learning-based Metaheuristics.

* Corresponding authors:
Maryam Karimi Mamaghan, E-mail: maryam.karimi@imt-atlantique.fr
Mehrdad Mohammadi, Tel: +33 2 29 00 10 30, E-mail: mehrdad.mohammadi@imt-atlantique.fr

Hub-and-Spoke Network Design under Congestion: A Learning-based Metaheuristic

Abstract

This paper models a single allocation multi-commodity hub-and-spoke network problem through a bi-objective mathematical model, considering the congestion in both hubs and connection links. A novel aggregation model is developed based on a general GI/G/c queuing system to evaluate the congestion of the flow of the multiple products in the hubs. The proposed model is then solved using a novel learning-based metaheuristic based on NSGA-II, *k*-Means clustering method, and an Iterated Local Search (ILS) algorithm. The proposed model and solution algorithm are validated through a set of experiments against optimal solutions, and benchmarked against four existing well-known algorithms.

Keywords: Hub-and-spoke network design; Bi-objective optimization; Congestion; Queuing network; Machine Learning; *k*-Means clustering method; Learning-based Metaheuristics.

1. Introduction

Hub-and-spoke systems have been largely employed in many-to-many transportation systems, wherein commodities (i.e., passengers, cargo parcels, telecommunication packets, etc.) are transferred between a many pairs of origin-destination (OD) nodes. Instead of directly connecting each pair of OD nodes, in a hub-and-spoke system, commodities are transferred through a set of intermediate nodes called hubs (Correia et al., 2018; Mohammadi et al., 2019a). Hubs serve the network as switching, transshipment, sorting, and distribution facilities. In a path from an origin to a destination, commodities from different origin nodes are consolidated at these hub facilities prior to be routed to an intermediate hub or to be delivered to their final destinations (Zhalechian et al., 2018; Taleizadeh et al., 2018, Zheng et al., 2019). The aggregation of commodities in hub facilities allows the exploitation of economies of scale due to the utilization of more efficient carriers with higher capacities on hub-to-hub connection links.

Different versions of hub-and-spoke systems have been proposed and studied in the literature differing from one another in several aspects as follows: 1) the allocation of each spoke to one and only one hub (i.e., single allocation) or to multiple hubs (i.e., multiple allocation), 2) the number and type of the located hubs, 3) the presence of absence of hub capacities (i.e., capacitated vs. uncapacitated), and 4) full connectivity (i.e., complete graph) or partial connectivity of hubs (i.e., incomplete graph) (Alumur and Kara, 2008; Dukkanci et al., 2019).

Although the exploitation of economies of scale is a big advantage of hub networks, this exploitation may lead to commodity overload in a small number of hubs, or even result in heavy-utilization/congestion of some hub-to-hub connections. This congestion becomes more critical for transportation companies that employ the hub-and-spoke network for shipment delivery where high delivery performance (i.e., low transportation time) is desired. Congestions in hub nodes and hub-to-hub connections highly affect the delivery performance, and hence, it is essential to take congestion effects into account when designing a hub-and-spoke network (de Camargo et al., 2009; de Camargo and Miranda, 2012; Mohammadi et al., 2019a; Rahimi et al., 2016).

More specifically, one of the main real applications of this study can be found in the national/worldwide shipment transportation system. In this system, different shipments with different priorities are consolidated in

the origin hubs. These shipments are then processed (e.g., packaged, loaded, etc.) and transited in larger volumes to different destinations through different hubs. Once arrived to the destination hub, shipments are again processed (e.g., unloaded, unpackaged, etc.) and sent to their final destinations. Since the capacity of hubs are limited, arrival shipments to each hub are congested and they should wait until they receive services. Such waiting time in the hubs significantly affect the performance of the network in term of transportation time between the moment that the shipment demand takes place until the moment that the shipment arrives to its final destination.

Three ways of addressing congestion in hub-and-spoke systems are 1) imposing classical capacity constraints on hub nodes for limiting the amount of flow entering the hubs, 2) incorporating costs of congestion effects explicitly into the objective function using a convex cost function that increases exponentially as more flows go through the hubs, and 3) accounting for congestion in hub nodes using queuing theory, and calculating the waiting time of the flow in terms of congestion.

The third approach has recently attracted more attention (Ishfaq and Sox, 2012; Mohammadi et al., 2017, 2019a) since it accounts for the congestion in the hubs in a more reliable and real way. In this regard, the most common way has been modelling the hubs as queuing system with arrival and/or service rates with Poisson distributions (i.e., M/M/c and M/D/c queuing systems). However, the stochasticity nature of the demand between OD nodes imposes a high degree of uncertainty on the inter-arrival time of the products. On the other hand, high variety of demands results in varying service rate of the products. Accordingly, employing general GI/G/c queuing system provides more reliable results (Ishfaq and Sox, 2012; Mohammadi et al., 2019b). Evaluating the congestion in the hubs when dealing with general GI/G/c queuing system become more complicated when dealing with multiple products with different characteristics (e.g., service rate, priority etc.).

In this regard, two real-world cases where the problem of this paper steps in are: 1) Shipment delivery system and 2) airway passenger transportation system. In the following, the first real-world application is explained in more detail. One of the most important shipment delivery systems is the mail or post system for physically transporting postcards, letters, and parcels. These shipments have different priorities (e.g., high priority, medium priority and low priority) and each shipment is processed based on its priority. In this system, different shipments with different priorities are first consolidated in the origin hub which is normally the center of each province in the country. These shipments are then processed (e.g., packaged, loaded, etc.) and transited in larger volumes to different destinations through different hubs and maybe with different transportation modes. Once arrived at the destination hub, shipments are again processed (e.g., unloaded, unpackaged, etc.) and sent to their final destinations. Since the capacity of hubs are limited, arrival shipments to each hub are congested and they should wait until they receive services. Such waiting time in the hubs significantly affects the performance of the network in terms of transportation time between the moments that the shipment demand takes place until the moment that the shipment arrives to its final destination. An efficient way to evaluate this congestion in the hubs is using queuing theory. On the other hand, the service time and inter-arrival time of shipments are stochastic and finding an exact distribution for them is hard (if not impossible). Accordingly, using typical queuing systems with exponential inter-arrival time and Poisson service rate are not practical enough. Therefore, general queuing systems (e.g., GI/G/1, GI/G/c) are the most efficient way to cope with this congestion evaluation. At each hub, the processing of shipments (e.g., unloading, sorting, packaging and loading) are done through different parallel working centers. For example, once trucks that transport the shipments arrive at the

hubs, they are referred to different ports (servers) of unloading and the shipment processes (unloading and sorting) are done in parallel. Since a wide variety of shipments may exist in each truck, service time at each server does not follow a unique and specific distribution. On the other hand, the arrival time of shipments to the hubs does not follow a specific stochastic distribution. Therefore, the service time and inter-arrival time of shipments should follow general distribution and using general queuing systems is the most efficient way to take into account the congestion of shipments in the hubs.

This paper addresses a bi-objective multi-product single allocation hub-and-spoke network design problem, wherein congestion is considered in both hubs and connection links. A GI/G/c queuing system is developed to evaluate the waiting time of the hubs. Finally, an aggregation model is proposed to handle multiple products in the GI/G/c queuing system. The first objective function of the model minimizes the sum of installation, allocation, and transportation costs, and the second objective function of the model minimizes the maximum transportation time between each pair of OD nodes that is significantly affected by congestion in the hubs and connection links.

Finally to solve the proposed model for large-sized instances, this paper develops a new hybrid metaheuristic algorithm based on non-dominated sorting genetic algorithm-II (NSGA-II) and a learning-based Iterated Local Search (L-ILS) algorithm. The idea of this hybridization is to develop an algorithm that is powerful in terms of both diversification (global search) and intensification (local search) and intelligently learns information during the searching process.

The rest of the paper is organized as follows. The most relevant paper considering the congestion in the hub are reviewed in Section 2. A bi-objective mathematical model is proposed in Section 3. Section 4 develops a hybrid metaheuristic algorithm to solve the proposed bi-objective mathematical model. The performance of the proposed hybrid algorithm is validated in Section 5 through numerous experiments. Section 6 deals with comprehensive sensitivity analysis of the model. Finally, the paper is concluded in Section 7.

2. Literature review

This section reviews the most relevant papers addressing congestion in the hub location problem.

2.1. Addressing congestion through capacity constraint

Although numerous papers have addressed the congestion effects by restricting the amount of flow entering a hub using capacity constraints, few of them have had the purpose of addressing congestion in the hub network. Accordingly, this part reviews those papers that initially aim at addressing congestion through capacity constraint.

As the first efforts, Grove and O'Kelly (1986) addressed the impact of congestion on hub-and-spoke networks. They showed how the delays in the schedules of airline systems are affected by the amount of flow entering the hubs. Marianov and Serra (2003) are among the first researchers that accounted for congestion in hub-and-spoke networks using queuing theory. They modeled the hub network as an M/D/c queuing system and proposed capacity constraints based on the waiting probability of flows in the hubs. They have also proposed a model for allocating servers to each installed hub. A similar work has been proposed by Rodriguez et al. (2007), wherein each hub is modeled with simple M/M/1 queuing system. Costa et al. (2008) aim at controlling congestion by minimizing the total time required to process the flow entering each hub. For this aim, the authors propose a second objective function that minimizes the maximum processing time of flows at hubs. Mohammadi et al. (2011) have modeled each hub as M/M/c queuing system and proposed a probabilistic constraint to ensure

that the probability of entering flows to the hubs that wait in a queue is less than a threshold value for each hub. Rahimi et al. (2016) have accounted for congestion through a queuing system with finite queue capacity. Accordingly, the authors model each hub as M/M/1/K queue system, where the probability of the flow exceeding the capacity of the hub is tried to be minimized.

Yang and Chiu (2016) addressed the hub network design problem considering demand uncertainty and hub congestion effects. Authors formulated the problem as a two-stage stochastic program with recourse model. The proposed model provides a consistent set of hub locations, while adjusting network configuration in response to different demand realizations. Özgün-Kibiroğlu (2019) addressed the hub location problem in which capacity restrictions were introduced into the objective function as a penalty cost to represent their congestion effects on respective hubs. The authors proposed a model allocating hubs whose capacities are larger than a desired level and still congestion on hubs is considered as a penalty cost in the objective function. Therefore, hubs are chosen between nodes which have higher capacities in order to reduce the penalty costs arising from surplus flows on hubs.

These works attempt to control congestion via limiting the flow entering hubs; however, merely controlling the amount of flow does not guarantee the higher performance of the hub network in terms of delivery time. Indeed, once the amount of flow approaches the capacity, congestion would happen in hub, although the capacity is still respected.

2.2. Addressing congestion through cost function

Modeling congestion through capacity constraint on the flows does not reflect the exponential nature of congestion effects: the more the flow into the hub, the harder the handling process. Consequently, greater costs are imposed to the hub network. Usually these costs increase extremely rapid due to congestion.

In this regard, Elhedhli and Hu (2005) have explicitly considered the congestion effect of each located hub as a cost term in the objective function for the hub-and-spoke problems. Using a power-law function which is widely utilized to estimate delay costs in airport applications, the authors propose a non-linear convex cost function formulation that increases rapidly as more traffic flows through the located hubs. Afterward, Elhedhli and Wu (2009) have proposed the same approach while considering each hub as an M/M/1 queue and used the Kleinrock's average delay function (Kleinrock, 2007) as a representation of the congestion effects.

Camargo and Miranda (2009) have studied a multiple allocation hub-and-spoke network design problem under hub congestion. They have proposed a non-linear mixed integer programming formulation, modeling the congestion as a convex cost function similar to Elhedhli and Hu (2005). In another work, Camargo and Miranda (2012) have addressed the hub-and-spoke network problem under congestion from two different network design perspectives; the network owner and the network user. The authors translate these two perspectives into mathematical programming models. The objective of the model is to minimize the installation, congestion, and routing costs over the network, wherein two different perspectives are analyzed: The network owner aims at designing network with the least cost, and the network user is willing to accept the minimum congestion effect at a reasonable cost. The proposed cost functions in the literature are disabled to evaluate the waiting time of the products, and they only account for the number of products accumulated in each hub. Evaluating the waiting time and the delay of products in the hubs are important when transport service providers aim at minimizing the total transportation time and attempt to offer the most competitive delivery services to their customers.

Kian and Kargar (2016) studied the hub location problem with a power-law congestion cost and propose an exact solution approach. Authors formulated this problem in a conic quadratic form and use a strengthening method which rests on valid inequalities of perspective cuts in mixed integer nonlinear programming. Alkaabneh et al. (2019) considered a hub-and-spoke network design problem with inter-hub economies-of-scale and hub congestion. They explicitly modeled the economies-of-scale as a concave piece-wise linear function and congestion as a convex function. The problem has been modeled as a nonlinear mixed integer program that is difficult to solve directly since the objective function has both convex and concave nonlinear terms and hence finding an optimal solution may not be easy. Finally, the authors proposed a Lagrangian approach to obtain tight upper and lower bounds.

2.3. Addressing congestion through waiting time calculation

A set of other works exist that address the congestion in the hub-and-spoke network, not via capacity constraint and congestion cost function, but by analyzing the waiting time resulted from the flow accumulation in the network. These works reflect the exponential nature of congestion effects in terms of the time spent by the products in the network.

In this regard, Ishfaq and Sox (2012) modeled the hubs as a GI/G/1 queuing systems and the shipments as multiple job classes with deterministic routings. By integrating the hub operation queuing model and the hub location-allocation model, the authors also investigated the effect of limited hub resources on the design of intermodal logistics networks under service time requirements. Mohammadi et al. (2017) have proposed a hub-and-spoke network for hazardous material (HAZMAT) transportation, wherein the congestion of HAZMAT in the hubs increases the risk of incidents. In addition, the risk increases more and more when the waiting time of the flows becomes longer and longer. Accordingly, the authors aim at minimizing the risk of HAZMAT congestion in the hubs by minimizing the waiting time of flows in the network. In their model, each hub is modeled as M/M/c queuing system where HAZMATs have different priorities, and HAZMATs with higher priority are served first.

As the most recent work, Mohammadi et al. (2019a) have designed a hub network by addressing congestion in both hubs and connection links. They consider each hub as M/M/1 queue system and model congestion at connection links via the Burea roads link performance function (Lo and Tung, 2003). The authors try to minimize the maximum transportation time between each pair of OD nodes. This time is affected by the congestion in the hubs as well as at the connection links.

Although these studies are able to evaluate the products' waiting time in the hubs, the considered queuing systems with Poisson arrival and service rates distributions (i.e., M/M/c and M/D/c queuing systems) are not able to fully capture the high variety of products and consequently high variation of the service time in the hubs. Accordingly, a general queuing system should be required to control the congestion of the hub-and-spoke network.

2.4. This paper's contributions

Table 1 summarizes the relevant literature of HLP in terms of modeling and solution approaches. Regarding the modeling approach, the reviewed papers are evaluated if they deal with: 1) single or multiple objectives, 2) congestion in the hubs and connection links, 3) classical or general queuing systems, and 4) aggregation of the flow in the hubs. Regarding the solution approach, the reviewed papers are classified whether

the authors develop 1) exact methods or metaheuristic algorithms as local search, global search or hybrid search algorithms and 2) learning-based search algorithms.

Table 1. Review of related work

Ref.	Year	Modeling approach							Solution Approach				
		Objective		Congestion		Queuing System							
		Single	Multiple	Hub	Link	Classical queue	General queue	Aggregation model	Exact Method	Local Search	Global search	Hybrid	Learning-based
O'Kelly	1986	√		√									
Marianov and Serra	2003	√		√		√				√			
Elhedhli and Hu	2005	√		√		√			√				
Rodriguez et al.	2007	√		√		√				√			
Costa et al.	2008		√	√		√			√				
Elhedhli and Wu	2009	√		√		√			√				
Camargo and Miranda	2009	√		√		√			√				
Camargo and Miranda	2012	√		√		√			√				
Ishfaq and Sox	2012	√		√			√			√			
Mohammadi et al.	2013	√		√		√					√		
Sedehzadeh et al.	2014	√		√		√					√		
Rahimi et al.	2016		√	√		√					√		
Yang et al.	2016	√		√							√		
Kian and Kargar	2016	√		√					√				
Mohammadi et al.	2017		√	√		√					√	√	
Azizi et al.	2018	√		√							√		
Hu et al.	2018	√			√		√						
Mohammadi et al.	2019b		√	√	√	√					√	√	
Özgün-Kibiroğlu et al.	2019	√		√							√		
Alkaabneh et al.	2019	√		√					√	√			
This paper			√	√	√		√	√		√	√	√	√

Based on Table 1, the main points that distinguish this work from the literature are listed below:

- Despite the literature, we study a multi-product version of the hub-and-spoke network design problem where multiple products are consolidated in the hubs and they need to be processed before being transferred toward their destinations.
- Almost all of the papers in the literature model the hubs as queuing system with arrival and/or service rates with Poisson distributions (i.e., M/M/c and M/D/c queuing systems), while the stochasticity nature of the demand between OD nodes imposes a high degree of uncertainty on the arrival rate of the products. On the other hand, high variety of demands results in varying service rate of the products. Accordingly, this paper develops a general GI/G/c queuing system to model the hubs.
- Since each hub processes multiple products simultaneously, products experience the same mean waiting time in each hub. Therefore, an aggregation model is required that considers all products and calculates

the overall waiting time in each hub. In this paper we consider for the first time a queue-based aggregation model for calculating the overall waiting time in a hub-and-spoke network.

- In addition to the contributions in the mathematical model, this paper proposes a new hybrid metaheuristic algorithm based on non-dominated sorting genetic algorithm-II (NSGA-II) and a learning based Iterated Local Search (L-ILS) algorithm. The idea of this hybridization is to develop an algorithm that is powerful in terms of both diversification (global search) and intensification (local search) and intelligently learns information during the searching process.
- Finally, sensitivity analyses are performed to investigate the performance of the network under stochastic disruption of the queuing systems as well as under the prioritization of the products.

3. Mathematical Model

In this section the congested bi-objective multi-product single allocation hub-and-spoke network problem under congestion is presented. Hereafter, this problem is simply called BiMSHC problem. In this problem, the goal is to locate *h* hubs in the network from H potential candidate locations. Then, a set of *N* spokes are allocated to located hubs. Each product *p* originating from a spoke is consolidated only in a single hub (i.e., single allocation). The graph of hubs is complete, meaning that each pair of located hubs are directly connected. Accordingly, the product flow between each pair of OD spokes, requires to pass through at least one hub (when both OD spokes are allocated to the same hub) or at most two hubs (when OD spokes are allocated to different hubs). Due to the limited capacity of the network, congestion happens in both 1) hubs that process the products by unloading, sorting, packaging and loading of products, and 2) connection links.

The rest of this section is organized as follows. Necessary notations are first presented in Section 2.1. Next, Section 2.2 presents the BiMSHC problem in terms of a mixed-integer non-linear programming (MINLP) model. Afterwards, Section 2.3 and 2.4 model the congestion in the connection links and the hubs, respectively. Finally, a piecewise function technique is proposed in Section 2.5 to linearize the proposed non-linear BiMSHC model.

3.1. Notations

Necessary notations are provided in this section.

Sets

N	Set of spokes
H	Set of potential hubs
P	Set of products

Indices

$i, j \in N$	Indices of spokes
$k, l \in H$	Indices of hubs
$p \in P$	Index of products

Parameters

f_k^p	Fixed cost of locating a hub at candidate node *k* to serve product *p*.
c_{ij}^p	Transportation cost of transferring product *p* between spokes *i* and *j*.
t_{ij}^p	Mean transportation time of transferring product *p* between spokes *i* and *j*.
w_{ij}^p	Flow of product *p* between spokes *i* and *j*.
μ_k^p	Service rate of processing product *p* at hub *k*.
$\tau_{S,k}^p$	Service time of processing product *p* at hub *k* (i.e., $\tau_{S,k}^p = 1/\mu_k^p$) . "*S*" stands for service.
$c_{A,pi}^2$	Squared coefficient of variation (SCV) of the inter-arrival time of product *p* from spoke *i*. "*A*" stands for arrival.
$c_{A,pk}^2$	SCV of the inter-arrival time of product *p* at hub *k*.

$c^2_{S,pk}$ SCV of the service time of product p at hub k.
Q_{kl} Capacity of link between hubs k and l for transferring the products. "S" stands for service.
c_k Number of servers at hub k.
h Number of hubs to be located in the network.
α_C Cost discount factor of transferring flow between hubs (i.e., $0 < \alpha_C < 1$).
α_T Time discount factor of transferring flow between hubs ($0 < \alpha_T < 1$).

Decision variables

X^p_{ik} 1 if spoke i is allocated to hub k for product p; 0 otherwise.
Z_k 1 if a hub is located at candidate node k; 0 otherwise.
X^p_{iklj} 1 if the flow of product p between spokes i and j passes first through hub k then hub l.
T^p_{iklj} Variable transportation time of transferring product p originated from spoke i with destination to spoke j passing first through hub k then hub l.
T^p_{kl} Variable transportation time of transferring product p from hub k to hub l.
$\lambda^p_{A,k}$ Flow of product p arriving at hub k.
W^p_k Variable waiting time of processing product p at hub k.
F^p_{kl} Amount of flow of product p traversing the link between hub k and l in both directions.

3.2. Proposed MINLP Model

Based on the notations provided in Section 3.1, the proposed mixed-integer non-linear programming model, to present the BiMSHC problem, is proposed as follow. Consider $c^p_{iklj} = c^p_{ik} + \alpha_C c^p_{kl} + c^p_{lj}$.

$$\mathbb{Z}_1 = \min \sum_{p=1}^{P}\sum_{i=1}^{N}\sum_{k=1}^{H}\sum_{l=1}^{H}\sum_{j=1}^{N} w^p_{ij} c^p_{iklj} X^p_{iklj} + \sum_{p=1}^{P}\sum_{k=1}^{H} f^p_k Z_k \quad (1)$$

$$\mathbb{Z}_2 = \min \max_{i,j,k,l,p} \{T^p_{iklj}\} \quad (2)$$

s.t.

$$\sum_{k=1}^{H} Z_k = h \quad (3)$$

$$\sum_{k=1}^{H}\sum_{l=1}^{H} X^p_{iklj} = 1 \qquad \forall i,j,p \quad (4)$$

$$X^p_{iklj} \le Z_k \qquad \forall i,j,k,l,p \quad (5)$$

$$X^p_{iklj} \le Z_l \qquad \forall i,j,k,l,p \quad (6)$$

$$X^p_{iklj}, Z_k \in \{0,1\} \qquad \forall i,j,k,l,p \quad (7)$$

Objective function (1) minimizes the sum of total transportation cost and the fixed cost of locating the hubs. Objective function (2) minimizes the maximum transportation time between each pair of OD nodes; wherein T^p_{iklj} is the transportation time of transferring product p originated from spoke i with destination at spoke j passing first through hub k then hub l. Constraint (3) determines the number of hubs to be located in the network. Constraint (4) ensures the single allocation of the spokes to the hub nodes for each product. Constraints (5) and (6) guarantee that the routes between OD nodes only traverse located hubs. Finally, constraint (7) is the integrality constraint for the decision variables.

Objective function (2) is not a linear function; therefore, its linear form is provided through a new objective function (8) and constraint (9), wherein $\mathbb{T}$ is the maximum transportation time between each pair of OD nodes.

$$\min \mathbb{T} \quad (8)$$

$$T^p_{iklj} \le \mathbb{T} \qquad \forall i,j,k,l,p \quad (9)$$

In constraint (9), the variable T_{iklj}^{p} is the sum of the transportation time on the route from spoke *i* to spoke *j* as well as the waiting time of the products at the hubs *k* and *l*. Accordingly, T_{iklj}^{p} is calculated as Equation (10).

$$T_{iklj}^{p} = \left(t_{ik}^{p} + W_{k}^{p} + \alpha_{T} T_{kl}^{p} + W_{l}^{p} + t_{lj}^{p}\right) X_{iklj}^{p} \qquad \forall i, j, k, l, p \tag{10}$$

The variable transportation time of product *p* between hubs *k* and *l*, T_{kl}^{p} , and the variable waiting time of product *p* at hub *k*, W_{k}^{p}, are calculated in Sections 3.3 and 3.4, respectively.

3.3. Variable inter-hub transportation time

A typical consideration in the hub-and-spoke literature is that the transportation time over the hub network is deterministic or stochastic but independent of the congestion of the connection links. In this section, we model the transportation between each pair of hub nodes (as the most congested connection links) as a function of a) capacity of the link and b) the amount of the flow traversing the link (Mohammadi et al., 2019a). For this aim, the Burea roads link performance function (Lo and Tung, 2003) is employed. It should be noticed that the mean transportation time of product *p* at hub *k* to hub *l*, T_{kl}^{p}, is equal for all products. Accordingly, the mean transportation time at hub-to-hub link *k* to *l*, T_{kl}, is formulated as Equation (11):

$$T_{kl}(F_{kl}, Q_{kl}) = t_{kl}\left[1 + \beta_{kl}\left(\frac{F_{kl}}{Q_{kl}}\right)^{\zeta_{kl}}\right] \tag{11}$$

where t_{kl} is link free-flow travel time between hubs *k* and *l,* and F_{kl} is the total flow of products traversing the link from hub *k* to hub *l* (i.e., $F_{kl} = \sum_{p} F_{kl}^{p}$). It is also considered that the transportation time is the same for all types of the products. As the congestion ratio F_{kl}/Q_{kl} increases, the transportation time of the link increases. The overload happens in the link if $\frac{F_{kl}}{Q_{kl}} > 1$ and no overload happens for cases where $0 < \frac{F_{kl}}{Q_{kl}} \leq 1$. In Equation (11), the constant and deterministic parameters β_{kl} and ζ_{kl} determine how the congestion ratio affects the transportation time. The parameter β_{kl} determines how the congestion ratio 'directly' affects the transportation time and the parameter ζ_{kl} is the shape of this effect that could be linear (ζ_{kl}=1), polynomial (ζ_{kl}=2) or with higher degrees ($\zeta_{kl} > 2$). For instance, if β_{kl}=1 and ζ_{kl}=2, the congestion ratio affects the transportation time over the link between hubs *k* and *l* in a polynomial manner (i.e., $T_{kl}(F_{kl}, Q_{kl}) = t_{kl}\left[1 + \left(\frac{F_{kl}}{Q_{kl}}\right)^{2}\right]$).

3.4. Variable waiting time at hub

In general, the products arriving in a hub have to wait in a queue if the hub is busy (Sedehzadeh et al., 2014). This waiting mainly depends on the rate at which the products arrive at the hub and the rate at which the hub processes the products. Since the inter-arrival time between the two arrivals and the service time of the hub are not deterministic and distributed around a mean value, it is essential to include the variability in time and its distribution while evaluating the performance indicators that account for the congestion in the hub (i.e., waiting time, queue length, throughput rate). Although most of the papers in the literature model the hubs as queuing system with arrival and/or service rates with Poisson distributions (i.e., M/D/c and M/M/c queuing systems), the stochasticity nature of the demand between OD nodes imposes a high degree of uncertainty on the arrival rate of the products. In addition, high variety of demands results in varying service rate of the products. Accordingly, we model each hub as a multi-server GI/G/c queue, wherein the inter-arrival times between flow units of each product *p* are given by a random variable with general distribution and mean $1/\lambda_{A,k}^{p}$. The service times are random variables with general distribution and mean $1/\mu_{k}^{p}$. At each hub, there exists a queue with infinite size, wherein the products are served in a first-come-first-served (FCFS) rule. At hub *k*, c_k parallel servers process the

products and each server serves only one unit of each product at a time and devotes all of its resources to complete the service. The throughput rate of product p at hub k is $\rho_k^p = \lambda_{A,k}^p/(c_k\mu_k^p)$ (i.e., $\rho_k^p = \tau_{S,k}^p/(c_k\tau_{A,k}^p)$).

Since obtaining an exact value for a performance indicator for hubs modeled by GI/G/c queues is difficult if not impossible, an approximation technique is required. As the first time in the hub location problem, the fork-join analysis (Satyam and Krishnamurthy, 2008) is adopted to evaluate the performance of each hub separately. In this analysis, the arrival products are aggregated into a single product (called aggregated product $\mathbb{P}$) and the performance of the hub is evaluated for the aggregated product. The parameters for the aggregated product are approximated based on the parameters of the original products at each hub. Figure 1 illustrates hub k that processes a given set of products.

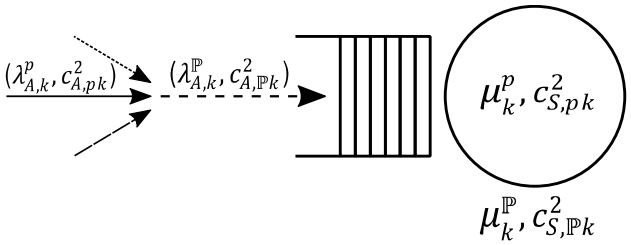


Figure 1. Hub k and aggregated product (Mohammadi et al., 2019b)

Based on the notations in Section 3.1, Equations (12) to (17) are proposed to estimate the parameters of the aggregated product (Mohammadi et al., 2018, 2019b).

$$\lambda_{A,k}^p = \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{l=1}^{H} w_{ij}^p X_{iklj}^p + \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{l=1}^{H} w_{ji}^p X_{jlki}^p \qquad \forall k,p \tag{12}$$

$$\lambda_{A,k}^{\mathbb{P}} = \sum_{p=1}^{P} \lambda_{A,k}^p \qquad \forall k \tag{13}$$

$$\tau_{S,k}^{\mathbb{P}} = \sum_{p=1}^{P} \left(\frac{\lambda_{A,k}^p}{\lambda_{A,k}^{\mathbb{P}}}\right)\tau_{S,k}^p \qquad \forall k \tag{14}$$

$$c_{A,pk}^2 = \vartheta_{pk}\gamma_{pk}^2 + (1-\vartheta_{pk}) \qquad \forall k,p \tag{15}$$

$$c_{A,\mathbb{P}k}^2 = \theta_k\delta_k^2 + (1-\theta_k) \qquad \forall i \tag{16}$$

$$c_{S,\mathbb{P}k}^2 = \frac{\left\{\sum_{p=1}^{P}\left(\frac{\lambda_{A,k}^p}{\lambda_{A,k}^{\mathbb{P}}}\right)\left(c_{S,pk}^2+1\right)\left(\tau_{S,k}^p\right)^2\right\} - \left(\tau_{S,k}^{\mathbb{P}}\right)^2}{\left(\tau_{S,k}^{\mathbb{P}}\right)^2} \qquad \forall k \tag{17}$$

where $\vartheta_{pk} = \left[1+4\left(1-\rho_{pk}\right)^2(v_{pk}-1)\right]^{-1}$, $\rho_{pk} = \sum_i\left(\frac{\sum_{j=1}^{N}\sum_{l=1}^{H} w_{ij}^p X_{iklj}^p + \sum_{j=1}^{N}\sum_{l=1}^{H} w_{ji}^p X_{jlki}^p}{\mu_k^p}\right)$, $\gamma_{pk}^2 = \sum_i\left(\frac{\sum_{j=1}^{N}\sum_{l=1}^{H} w_{ij}^p X_{iklj}^p + \sum_{j=1}^{N}\sum_{l=1}^{H} w_{ji}^p X_{jlki}^p}{\lambda_{A,k}^p}\right)c_{A,pi}^2$ and ${v_{pk}}^{-1} = \sum_i\left(\frac{\sum_{j=1}^{N}\sum_{l=1}^{H} w_{ij}^p X_{iklj}^p + \sum_{j=1}^{N}\sum_{l=1}^{H} w_{ji}^p X_{jlki}^p}{\lambda_{A,k}^p}\right)^2$. In addition, $\theta_k = [1+4(1-\rho_{\mathbb{P}k})^2(v_k-1)]^{-1}$, $\delta_k^2 = \sum_p\left(\frac{\lambda_{A,k}^p}{\lambda_{A,k}^{\mathbb{P}}}\right)c_{A,pk}^2$ and ${v_k}^{-1} = \sum_p\left(\frac{\lambda_{A,k}^p}{\lambda_{A,k}^{\mathbb{P}}}\right)^2$. After calculating the parameters of the aggregated product $\mathbb{P}$ at hub k, the waiting time of product p in the queue at hub k, W_k^p, is provided as Equation (18), where in $\rho_k^{\mathbb{P}} = \lambda_{A,k}^{\mathbb{P}}\tau_{S,k}^{\mathbb{P}}/c_k$. The main point is that all the products entering a particular hub, where there is no priority between them, experience the same waiting time in that hub.

$$W_k^p \approx W_k^{\mathbb{P}} \approx \frac{\phi_k^{\mathbb{P}}\tau_{S,k}^{\mathbb{P}}}{1-\rho_k^{\mathbb{P}}} \times \frac{c_{A,\mathbb{P}k}^2 + c_{S,\mathbb{P}k}^2}{2} \times G_k^{\mathbb{P}} \qquad \forall k,p \tag{18}$$

where $\phi_k^{\mathbb{P}}$ and $G_k^{\mathbb{P}}$ are calculated as Equations (19) and (20):

$$\phi_k^{\mathbb{P}} = \frac{\left(c_k\rho_k^{\mathbb{P}}\right)^{c_k}}{c_k!\left(1-\rho_k^{\mathbb{P}}\right)}\left[\sum_{n=0}^{c_k-1}\frac{\left(c_k\rho_k^{\mathbb{P}}\right)^n}{n!}+\frac{\left(c_k\rho_k^{\mathbb{P}}\right)^{c_k}}{c_k!}\frac{1}{1-\rho_k^{\mathbb{P}}}\right]^{-1} \qquad \forall k \tag{19}$$

$$G_k^{\mathbb{P}} = \begin{cases}\exp\left(-\frac{2}{3}\times\frac{1-\rho_k^{\mathbb{P}}}{\rho_k^{\mathbb{P}}}\times\frac{\left(1-c_{A,\mathbb{P}k}^2\right)^2}{c_{A,\mathbb{P}k}^2+c_{S,\mathbb{P}k}^2}\right), & 0\le c_{A,\mathbb{P}k}\le 1\\ \exp\left(-\left(1-\rho_k^{\mathbb{P}}\right)\times\frac{c_{A,\mathbb{P}k}^2-1}{c_{A,\mathbb{P}k}^2+c_{S,\mathbb{P}k}^2}\right), & c_{A,\mathbb{P}k}>1\end{cases} \qquad \forall k \tag{20}$$

3.5. Linearization of the MINLP model: A piecewise function

The proposed model in Section 3.2 along with the variable transportation time and approximated waiting time of the Sections 3.3 and 3.4 is non-linear. Accordingly, finding an optimal solution even for some small-size instances of the problem might be impossible or very time-consuming. In this section, we employ a piecewise linear approximation technique (Mohammadi et al., 2019a) to approximately linearize the non-linear terms T_{kl} and W_k^p.

Let *f*(*a*) be the piecewise linear approximation of a single variable by considering *S* number of sampling coordinates $a_1,\dots,a_S$ on the *s* axis (*breakpoints*), on which the function is evaluated. The function is then approximated by the linear term $\left[\left(a_s,f(a_s)\right),\left(a_{s+1},f(a_{s+1})\right)\right]$ $(s=1,\dots,S-1)$. Therefore, for any given *a* value, where $a_s\le\bar{a}\le a_{s+1}$, the function value $f^{\approx}(\bar{a})$ is approximated as Equations (21) and (22).

$$\bar{a}=\omega a_s+(1-\omega)a_{s+1} \tag{21}$$

$$f^{\approx}(\bar{a})=\pi f(a_s)+(1-\pi)f(a_{s+1}) \tag{22}$$

where π is a (unique) value in [0,1].

To use this linearization technique, a set of new variables and constraints should be defined to associate any *a* value to the proper pair of consecutive breakpoints. Since T_{kl}^p and W_k^p are non-linear variables, two piecewise linear approximations are needed. Consider a continuous variable v_s for each breakpoint *s*, where $v_s\in[0,1]$; $(s=1,\dots,S)$. In addition, u_s is a binary variable associated with the *s*th interval $[a_s,a_{s+1}]$ $(s=1,\dots,S-1)$, where $u_0=u_s=1$ at the extremes. Finally, the approximate value $f^{\approx}$ can be obtained by imposing the following constraints and introducing special v_s and u_s corresponding to each nonlinear term. Required notations are first presented and additional constraints are then presented to linearize the non-linear terms of objective function (2).

Parameters:

$\lambda_{A,k,\pi}^{\mathbb{P}}$	πth breakpoint of the total flow arriving at hub *k* ($\lambda_{A,k}^{\mathbb{P}}$).
$F_{kl,\pi}$	πth breakpoint of the total flow traversing the link *k* to *l* (F_{kl}).
$W\left(\lambda_{A,k,\pi}^{\mathbb{P}}\right)$	Waiting time corresponding to the total flow $\lambda_{A,k,\pi}^{\mathbb{P}}$ arriving at hub *k*.
$T\left(F_{kl,\pi}\right)$	Transportation time corresponding to the total flow $F_{kl,\pi}$ traversing the link *k* to *l*.
G	An arbitrary large number.

Decision variables:

$v_s^{W,k}$	Continuous variable for breakpoint π associated with $W\left(\lambda_{A,k,\pi}^{\mathbb{P}}\right)$.
$v_s^{T,kl}$	Continuous variable for breakpoint π associated with $T\left(F_{kl,\pi}\right)$.
$u_s^{W,k}$	Binary variable for breakpoint π associated with $W\left(\lambda_{A,k,\pi}^{\mathbb{P}}\right)$.
$u_s^{T,kl}$	Binary variable for breakpoint π associated with $T\left(F_{kl,\pi}\right)$.

Constraints (23) to (34) are provided to linearly approximate the non-linear terms T_{kl} and W_k^p in the proposed model.

$$\sum_{s=1}^{S-1} u_s^{W,k} = 1 \qquad \forall k \tag{23}$$

$$v_s^{W,k} \le u_{s-1}^{W,k} + u_s^{W,k} \qquad \forall k, s \in S \tag{24}$$

$$\sum_{s=1}^{S} v_s^{W,k} = 1 \qquad \forall k \tag{25}$$

$$\lambda_{A,k}^{\mathbb{P}} = \sum_{s=1}^{S} v_s^{W,k}\, \lambda_{A,k,s}^{\mathbb{P}} \qquad \forall k \tag{26}$$

$$W_k^{\mathbb{P}} = \sum_{s=1}^{S} v_s^{W,k}\, W\left(\lambda_{A,k,s}^{\mathbb{P}}\right) \qquad \forall k \tag{27}$$

$$\sum_{s=1}^{S-1} u_s^{T,kl} = 1 \qquad \forall k, l \tag{28}$$

$$v_s^{T,kl} \le u_{s-1}^{T,kl} + u_s^{T,kl} \qquad \forall k, l, s \in S \tag{29}$$

$$\sum_{s=1}^{S} v_s^{T,kl} = 1 \qquad \forall k, l \tag{30}$$

$$F_{kl} = \sum_{s=1}^{S} v_s^{T,kl}\, F_{kl,s} \qquad \forall k, l \tag{31}$$

$$T_{kl} = \sum_{s=1}^{S} v_s^{T,kl}\, T\left(F_{kl,s}\right) \qquad \forall k, l \tag{32}$$

$$v_s^{W,k}, v_s^{T,kl} \in [0,1] \qquad \forall k, l, s \in S \tag{33}$$

$$u_s^{W,k}, u_s^{T,kl} \in \{0,1\} \qquad \forall k, l, s \in S \tag{34}$$

Finally, the final proposed mixed-integer linear programming model is presented as follow:

$$\mathbb{Z}_1 = \min \sum_{p=1}^{P}\sum_{i=1}^{N}\sum_{k=1}^{H}\sum_{l=1}^{H}\sum_{j=1}^{N} w_{ij}^p c_{iklj}^p X_{iklj}^p + \sum_{p=1}^{P}\sum_{k=1}^{H} f_k^p Z_k \tag{1}$$

$$\mathbb{Z}_2 = \min \mathbb{T} \tag{8}$$

$$\left(t_{ik}^p + W_k^p + \alpha_T T_{kl}^p + W_l^p + t_{lj}^p\right) X_{iklj}^p \le \mathbb{T} \qquad \forall i, j, k, l, p$$

s.t.: Constraints (3)-(7), (23)-(34)

4. Hybrid metaheuristic solution algorithm

Due to the complexity of the hub-and-spoke network design problem to find the optimal solution (Alumur and Kara, 2008), metaheuristic algorithms have been widely proposed to solve this problem (Rodríguez et al., 2007; Mohammadi et al., 2016; Rahimi et al., 2016; Mohammadi et al., 2017; Zhalechian et al., 2018; Mohammadi et al., 2019a). Among them, hybrid metaheuristic algorithms, as the combination of evolutionary algorithms (EAs) and local search (LS) algorithms, have attracted higher attention for their outperformance in terms of both diversification and intensification of the solution space (Mohammadi et al., 2019a).

In recent years, the combination between evolutionary algorithms (EAs) and machine learning (ML) and has received considerable attention from the research community. Interested readers are referred to (Jourdan et al., 2006; Zhang et al., 2011; Calvet et al., 2017) and references therein. In this regard, this section develops a hybrid metaheuristic solution algorithm to solve the proposed BiMSHC model based on Non-dominated Sorting Genetic Algorithm-II (NSGA-II) (Deb et al., 2000) and a Learning-based Iterated Local Search (L-ILS)

algorithm. The well-known 𝕜-means clustering algorithm (Likas et al., 2003) is used as the machine learning technique. For simplicity, the proposed hybrid algorithm is then called GAMLS.

As a population-based EA, NSGA-II can present a set of non-dominated Pareto (NDP) optimal solutions of the multi-objective optimization problems (MOOPs) with a set of conflicting objective functions (Deb et al., 2000). NSGA-II evolves a population of randomly generated solutions with size P_{size} using elitism, crossover and mutation operators. Though elitism, a percent E_R (elitism rate) of the best solutions are directly transferred to the next generation. The rest of the population of the new generations are filled by crossover and mutation operators with rates C_R and M_R, respectively, where $E_R + C_R + M_R = 1$. In crossover, a pair of selected parents (using tournament selection mechanism) are crossed with the hope of creating better children. Despite crossover, mutation operator is employed on an individual solution in order to diversify the solution space and escape from the local optimum. These operators are employed to generate new solutions as a next offspring so that the current population and generated offspring are combined together for creating next generation according to non-dominance and crowding distance (CD) techniques. The CD is a measure of the density of solutions. The value of the CD presents an estimate of the density of solutions surrounding a particular solution. Accordingly, a solution with higher CD value is preferred. The idea behind the CD is that isolated solutions with high CD value belong to undiscovered regions and keeping these solutions in the population increases the diversification capability of the algorithm. This evolution process continues until a maximum iteration Itr_{max}^{GA} is reached.

ILS is a well-known metaheuristic algorithm for solving NP-hard optimization problems due to its effectiveness in both intensification and diversification as well as its simplicity in practice. When a search is trapped in a local optimum, ILS helps the search to escape the trap without losing many of the good properties of the current solution. The main iteration loop of the ILS algorithm does three main following steps (Karimi-Mamaghan et al., 2020): 1) ILS algorithm performs a *Perturbation*(.) function over the current local optimum solution s_i^* to help the search to escape the local minimum; whereby an intermediate solution s_i' is generated. Through this random perturbation, the local optimum solution s_i^* can be transferred to another place in the solution space, 2) After applying the *Perturbation*(.) function, the *LocalSearch*(.) function is performed on the intermediate solution s_i' to obtain a new local optimal solution $s_i^{*\prime}$, and 3) After applying the *LocalSearch*(s_i') function, *AcceptanceCriterion*(s_i^*,$s_i^{*\prime}$,s_{best}) is employed to check whether to accept the current local optimal solution s_i^* comparing to the s_i^*. The *AcceptanceCriterion*(.) function can only accept better solution or it can even accept worse solution with a small gap (e.g., 5% gap from s_i^*). It is furtherly checked if s_{best} can be also updated. These three steps continue until a maximum iteration Itr_{max}^{ILS} is reached.

4.1. Proposed hybrid GAMLS algorithm

In hybrid algorithms for solving MOOPs, three main issues need to be resolved concerning the local search mechanism: 1) which solutions to initiate the local search, 2) how to select solutions from a set of candidate solutions, and 3) how to perform the local search on the selected solutions in order to escape from the local optimum and find better solutions. Regarding the first issue, the set of NDP solutions are more preferable to be the candidate to perform local search (Chiang et al., 2011; Wang and Tang, 2017). Sometimes, the number of NDP solutions are too high and performing a local search over each of them is computationally inefficient (if not impossible). But it should be noted that the number of initial solutions should be also enough to have a good diversification. So a set of representative solutions among the NDP solutions to initiate the local searches are necessary. For the third issue, the local search algorithm should be not only powerful in intensification, but also

performant enough in terms of diversification. In addition, an efficient mechanism should exist to escape from the local optimum.

Based on this motivation, the hybrid GAMLS algorithm is proposed to handle these three issues. The NSGA-II, as a powerful population-based EA, is responsible to obtain NDP solution to initiate the ILS. In the meantime, 𝕜-means clustering algorithm is used as the link between NSGA-II and ILS to select the representative solutions from the results of NSGA-II and to feed them to ILS algorithm.

Once the NSGA-II obtains the NDP solutions, the 𝕜-means algorithm is employed to find 𝕜 representative solutions (Repst. Sol) from the NDP set. Actually, the 𝕜-means clustering algorithm is employed over the solution representation of the solutions. The 𝕜-means algorithm creates 𝕜 separate solution clusters, wherein the solutions in each cluster are closed together and probably from the same region of the solution space (i.e., same local optimum); while the solutions of a cluster are probably far from the other clusters' solutions (i.e., different local optimums). One may create clusters based on the objective function values of the solutions but it does not guarantee that the solutions in one cluster are closed together. Indeed, two solutions may have the same objective function but they belong two different regions of the solution space. Afterward, the solution at the center of each cluster or the closest solution to the center of the cluster will represent the whole cluster. The closeness of two solutions is defined as the minimum difference between the solution representations of those two solutions. Finally, these 𝕜 representative solutions are fed to the ILS algorithm and their neighborhood is intensified. At the end, the new found NDP solutions via ILS executions are combined with those of NSGA-II and the final NDP solutions are returned. Figure 2 depicts the overall mechanism of the hybrid GAMLS algorithm.

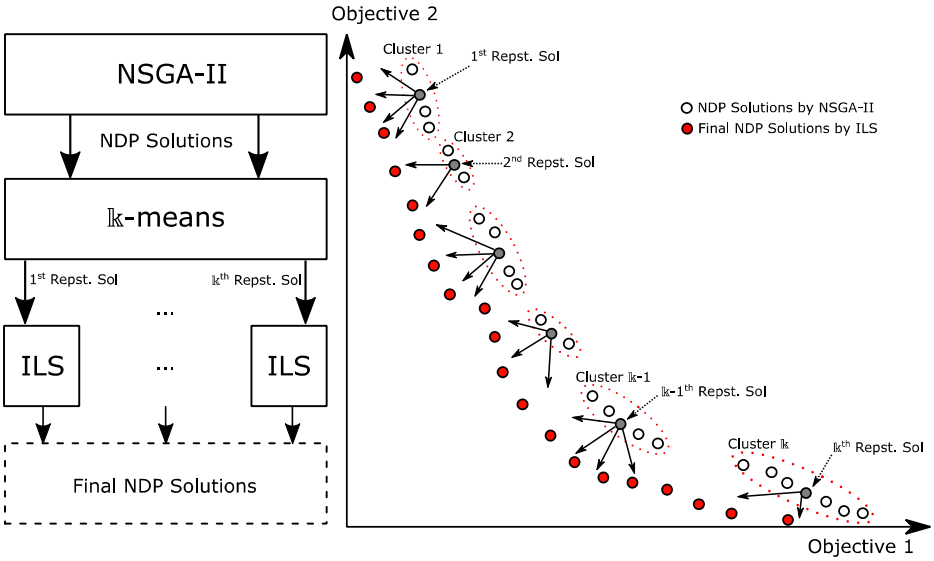


Figure 2. Mechanism of the hybrid GAMLS algorithm

4.2. Solution representation

The solution vector representation in every EA should be as compact as possible but should contain enough information to represent any solution of the problem. The way of representing solutions significantly affect the choice of searching operators. Accordingly, efficient representation of the solutions helps to use well-known operators in the literature that their high performance has been proved. An important recommendation is that represent the solution in a way that applying operators do not possibly lead to infeasible solutions. But if a

solution vector is infeasible and violates one or certain number of constraints, a penalty value can be added to its corresponding objective function values. This penalty value decreases the chance of that solution to participate in the reproduction procedures. Generally, it is much better to propose a representation that only includes feasible solutions.

In this paper, a solution is represented as a (P×N) matrix where the arrays are the h values from {1,…,H}. Figure 3 depicts a hub-and-spoke network with P=2 products, N=8 spokes, and h=3 hubs among a set of H=6 possible candidate locations. It can be seen that spokes 1 to 8, transfer product 1 to hubs 3, 3, 1, 5, 1, 5, 5, and 3, respectively. For product 2, the assignment of spokes is to hubs 5, 1, 1, 3, 3, 5, 1, and 3, respectively. Figure 4 depicts the corresponding hub-and-spoke network of Figure 3.

	Spokes							
	1	2	3	4	5	6	7	8
Product 1	3	3	1	5	1	5	5	3
Product 2	5	1	1	3	3	5	1	3

Figure 3. Solution representation of a multi-commodity hub-and-spoke network

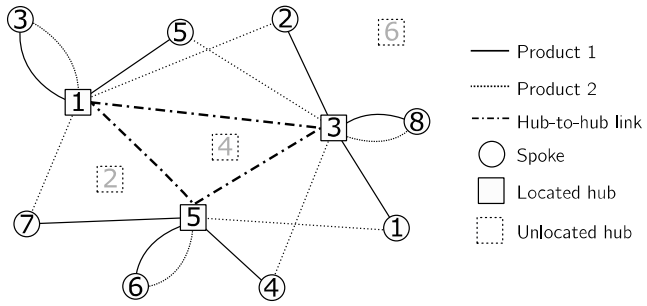


Figure 4. Hub-and-spoke network design of Figure 3

Having the structure of the hub-and-spoke network (e.g., Figure 3 and 4), the objective functions of the proposed model can be easily calculated. The first objective function is calculated based on the decision variables that shows which spokes have been allocated to which hubs and which hubs have been located in the network. These decision variables can be calculated through the structure of the network determined by the solution representation. For the second objective function, two terms should be calculated, the mean transportation time at hub-to-hub link k to l, T_{kl}, and the waiting time in hub k for product p, W_k^p. In these terms of the second objective function, the amount of flow of products traversing the spoke-to-hub and hub-to-hub links should be first calculated. To calculate these amounts of flow, it should be noted first that each spoke i is allocated to only a single hub k (single allocation) for each product p. Next, the flow of product p over the connection link from spoke i to hub k is equal to the total flow originating from spoke I ($\sum_j w_{ij}^p$). The amount of flow between each pair of hubs k and l (F_{kl}) can be simply calculated based on the spokes allocated to these two hubs k and l and the corresponding flow between each pair of the spokes. Having the amounts flow traversing the hub network, the mean transportation time at hub-to-hub link k to l, T_{kl}, and the waiting time in hub k for product p, W_k^p are calculated by equations (11) and (18)-(20), respectively.

4.3. Crossover, mutation & local search operators

For doing crossover on two selected parents, two types of crossover operators are used as *one-point* crossover and *two-point* crossover (Mohammadi et al., 2013). In *one-point* crossover, two parents are crossed from one cutting point across the columns of the solution matrix. In *two-point* crossover, two parents are crossed

from two cutting point across the columns of the solution matrix. At each call of crossover function, one operator is employed randomly. For employing mutation on a single solution, two types of mutation operators are used as *reversion* and *random generation*. In the *reversion* operator, two columns far enough each other are selected and the permutation of the in-between columns are reversed. In the *random generation* mutation, a new solution is generated randomly. Similar to crossover, mutation function uses one operator at each call of doing mutation.

In the ILS algorithm, local search operators are *swap*, *re-allocation* and *hub-removal* operators. In a *swap* operator, two adjacent arrays at each row of the solution matrix are swapped. In *re-allocation*, a spoke is selected at each row of the solution matrix and it is re-allocated to an existing hub. Finally, in *hub-removal* operator, a hub is selected from the network and is replaced by a non-existing hub. The perturbation operator of ILS is a consecutive employment of *reversion* and *hub-removal* operators.

Indeed, the crossover operator may create infeasible solutions that violate Constraint (3). In this situation, two cases happen: 1) the number of located hub is less than h or 2) the number of located hubs is greater than h. For the first case, a penalty is added to the objective value of that solution. In the second case, the redundant hubs are eliminated from the solution and their spokes are allocated to their closest hubs. The redundant hubs are the hubs with higher fixed cost. Finally, the Pseudo code of the proposed GAMLS algorithm is as Algorithm 2.

Algorithm 2. GAMLS algorithm

Set parameters: Itr_{max}^{GA}, P_{size}, E_R, C_R, M_R, Itr_{max}^{ILS}, $\mathbb{k}$ // *k is the number of clusters in the $\mathbb{k}$-means algorithm*
$\mathbb{S} = \{\}$ // *Archive of NDP solutions*

NSGA-II Global Search Algorithm

Generate initial random population of size P_{size}
Evaluate the objective functions of each solution
Rank the solutions based on the non-dominance sorting // *1st ranked solutions are NDP solutions*
Calculate CD for each solution // *CD is calculated among the same-ranking solutions*
$\mathbb{S}$:= Update archive of NDP solutions via 1st ranked solutions
While Itr_{max}^{GA} not reached **Do**
  NG = { }
  NG = NG ∪ {Best E_R NDP solutions with high CD from $\mathbb{S}$} // *NG: Next Generation* // *Elitism*
  For C_R/2 times **Do**
    s_1, s_2 := Select two solutions via TS // *TS: Tournament Selection*
    o_1, o_2 := Perform crossover on s_1, s_2 // *Creating offspring using crossover operator*
    NG = NG ∪ {o_1,o_2}
  EndFor
  For C_M times **Do**
    s_3 := Select a solution randomly
    s_M := Perform mutation on s_3 // *Diversifying solutions using mutation operator*
    NG = NG ∪ {s_M}
  EndFor
  $\mathbb{S}$:= Update archive of NDP solutions
  Evaluate the objective functions of each solution
  Rank the solutions based on the non-dominance sorting
  Calculate CD for each solution
EndWhile

Machine Learning Algorithm

[NDP_1, …, $NDP_{\mathbb{k}}$] := Select $\mathbb{k}$ NDP solutions by $\mathbb{k}$-means

Iterated Local Search Algorithm

For s_i in [NDP_1, …, $NDP_{\mathbb{k}}$] **Do**
  s_i^* := *LocalSearch*(s_i)
  s_{best} := s_i^*
  While stopping criterion not reached **Do**
    s_i' := *Perturbation*(s_i^*)
    $s_i^{*\prime}$:= *LocalSearch*(s_i')
    s_i^* := *AcceptanceCriterion*(s_i^*,$s_i^{*\prime}$,s_{best})
  EndWhile
  $\mathbb{S}$:= Update archive of NDP solutions
EndFor

4.4. Time complexity of the proposed GAMLS algorithm

The time complexity of an algorithm is concerned about how fast or slow that algorithm performs and describes the amount of time it takes to run that algorithm. Time complexity is usually calculated by counting the number of elementary operations performed by the algorithm, supposing that each elementary operation takes a fixed amount of time to perform. Therefore, the amount of time taken by an algorithm is the number of elementary operations multiplied by a constant factor (Oliveto and Witt, 2015). Since the execution time of an algorithm may vary among different inputs of the same size, one usually considers the worst-case time complexity, which is the maximum amount of time required for inputs of a given size. The complexity of an algorithm is commonly shown as O(N), where N is the size of the problem.

The complexity of the proposed GAMLS of Algorithm 2 is the sum of the time complexity of three NSGA-II, 𝕜-Means and ILS algorithms. The time complexity of each part is described as follows:

- NSGA-II: The complexity of the NSGA-II is the sum of different parts as
    1) Population sorting of size P_{size}. Time complexity is $O(P_{size} \log P_{size})$,
    2) Fitness function evaluation that depends on the population size (P_{size}), number of products (P) and number of spokes (N). Time complexity is $O(P_{size} \times PN^2)$,
    3) Solution generating using crossover that includes both tournament selection (TS) and crossover (XO) operator with rate C_R. Time complexity is $O\left(\overbrace{(\overbrace{C_R \times P_{size}}^{XO})}^{TS}\right)$,
    4) Solution generating using mutation that includes the mutation (MO) operator with rate M_R. Time complexity is $O(M_R \times P_{size})$,
    5) The main loop of the NSGA-II with Itr_{max}^{GA} number of iterations.

  Finally, the complexity of the NSGA-II is equal to.
- 𝕜-Means: The complexity of the 𝕜-Means clustering algorithm in the proposed GAMLS algorithms is equal to $O(|\mathbb{S}| \times N \times \Bbbk)$, where $|\mathbb{S}|$ is the number of the NDP solutions obtained by the NSGA-II.
- ILS: The complexity of the ILS algorithm depends on the number of the NDP solutions ($|\mathbb{S}|$), local search operators' complexity and the Itr_{max}^{ILS} number of iterations. Time complexity of the ILS algorithm is equal to $O(Itr_{max}^{ILS} \times |\mathbb{S}| \times N)$.

5. Computational experiment

The performance of the proposed GAMLS algorithm is validated through a benchmark with three metaheuristic algorithms in the literature, classical NSGA-II (Deb et al., 2000) and two new recently developed hybrid algorithms that employ machine learning to improve the performance of the metaheuristic algorithms (Sun et al., 2019; Zhang et al., 2016). The classical NSGA-II algorithm is similar to GAMLS without performing 𝕜-means and ILS algorithms. The aim of this comparison is to discover how hybridizing the NSGA-II with machine learning and ILS algorithm improves the quality of the NDP solutions.

Sun et al. (2019) propose an adaptive MOEA for MOOPs. In their algorithm, a 𝕜-means clustering method is employed to learn the Pareto optimal set's manifold structure adaptively, in accordance with the regularity property of MOOPs, along the evolution. A new offspring is generated in two ways: either 1) a solution in each cluster is Gaussian-perturbed using the variance-covariance matrix within its cluster, or 2) a solution is generated using differential evolution (DE) operator from two parents and a global solution from different clusters. These two strategies make balance between diversification and intensification. Zhang et al. (2016) propose a self-organizing multi-objective EA (MOEA), wherein a self-organizing mapping (SOM) method with ($m-1$) latent variables is applied to establish the neighborhood relationship among current

solutions. Their method only allows the mate between neighboring solutions to generate a new solution. To reduce the computational overhead, the self-organizing training step and the evolution step are conducted in an alternative manner.

Although these algorithms have been proposed for optimization problems with continuous variables and differentiable objective functions, we adapt their idea and develop two new algorithms: one based on NSGA-II and 𝕜-means (Sun et al., 2019) and second one based on NSGA-II and self-organizing map (SOM) method (Zhang et al., 2016). These algorithms are, respectively, called *HEU1* and *HEU2* hereafter. The 𝕜-means and SOM are used to make clusters and the clusters are updated at each iteration of the algorithms if significant difference happens between the new generated solutions and those of the previous generation; otherwise, the most recent clusters are used in the new iteration of the algorithm. Once clusters are generated, new solutions are generated to search the solution space. When generating new solutions using crossover, two intra-cluster and inter-cluster search mechanisms are adapted. For intra-cluster mechanism, inside each cluster, two solutions are randomly selected using a binary tournament selection and one-point crossover operator (Section 4.3) is applied to create two new offspring. For inter-cluster mechanism, three solutions s_1, s_2 and s_3 are selected in such a way that s_2 and s_3 are from the same cluster and s_1, is the best solution in the population (in terms of dominance and crowding distance) but not in the same cluster as s_2 and s_3. Solution s_1 is named as the target solution. Afterward, solutions s_2 and s_3 are first crossed using one-point crossover (Section 4.3). Then, new created solution from s_2 and s_3 is crossed by the target solution s_1 again using one-point crossover. The idea of inter-cluster mechanism is not only to keep the similarity of solutions in a cluster but also to diversify the solutions between clusters. Inter-cluster mechanism is applied as many as the number of solutions in the population. For doing mutation, at each iteration, a solution is randomly selected from each cluster and reversion operator (Section 4.3) is employed.

All algorithms are compiled in Python 3 programming language executed on a Pentium 8 CPU with 3.4 GHz processor and 32 GB of RAM.

5.1. Design of experiments

This section design a set of 20 instances with three size categories to compare the performance of the proposed GAMLS algorithm with those of NSGA-II, HEU1 and HEU2. The three categories are created based on the computational time required to find the optimal Pareto solutions. Categories A and B consist of the instances that obtaining optimal Pareto solutions for them takes [82s, 2h] and [3h, 5h], respectively. Finally, category C includes the instances that cannot be solved to optimality in reasonable time (i.e., the augmented ϵ-constraint method cannot find even a single Pareto optimal solution in a reasonable time). Table 2 shows the property of the instances in terms of number of products P, number of spokes N, and number of hubs h. It is worth mentioning that the generation of these data has been inspired from classical instances in hub-and-spoke network design problem (i.e., CAB, AP, USA423, Turkish network). Actually, this inspiration has been only to get the idea on the range of input parameters such as flow and cost. In addition, the transportation time between each pair of nodes has been calculated based on the distance between cities. On the other hand, since the CAB, AP and other data sets do not contain other parameter that our model requires, these parameters have been generated randomly but based on real assumption. For example, hubs with higher fixed cost have higher performance when serving the arrival flow. Therefore, the CAB, AP and other data sets could not be used directly in the proposed model. Finally, we consider that $\alpha_C = 0.75$, $\alpha_T = 0.70$, $\beta_{kl}=1$ and $\zeta_{kl}=2$.

It is obvious the performance of an algorithm is significantly affected by the value of its parameters. Higher performance of each algorithm can be obtained by appropriate tuning of its parameters using response surface methodology (RSM) (Azizmohammadi et al., 2013; Mohammadi et al., 2014). RSM is defined as a collection of mathematical and statistical method-based experiments, which can be used to optimize processes. Regression equation analysis is used to evaluate the response surface model. To tune the parameters, two levels for each parameter are considered. Each factor is measured at two levels, which can be coded as −1 when the factor is at its low level ($\mathbb{L}$) and +1 when the factor is at its high level ($\mathbb{H}$). The coded variable can be then defined as $x_i = \left(r_i + 0.5(\mathbb{h} + \mathbb{l})\right)/0.5(\mathbb{h} - \mathbb{l})$, where x_i and r_i are coded as variable and real variable, respectively. $\mathbb{h}$ and $\mathbb{l}$ represent high level and low level of the factor. Using RSM, tuned parameters of the GAMLS, NSGA-II, HEU1 and HEU2 algorithms are considered as follows:

Table 2. Problem instances to evaluate the performance of GAMLS algorithm

Category	Instance	Parameters							
		P	*N*	*h*	w^* (unit/h)		c^*	μ^* (unit/h)	
					mean	SCV		mean	SCV
Cat. A	I1	2	6	2	[10,30]	[1,5]	2	[40,80]	[1,5]
	I2	2	8	3	[30,50]	[4,8]	2	[60,100]	[4,8]
	I3	3	8	2	[50, 80]	[6,10]	3	[100,180]	[6,10]
	I4	3	10	4	[80,120]	[8,12]	3	[150,250]	[8,12]
	I5	4	10	3	[10,30]	[1,5]	2	[100,180]	[1,5]
	I6	5	10	4	[30,50]	[4,8]	3	[60,120]	[4,8]
Cat. B	I7	5	12	4	[50, 80]	[6,10]	3	[200,240]	[6,10]
	I8	6	12	5	[80,120]	[8,12]	4	[220,300]	[8,12]
	I9	6	14	3	[10,30]	[1,5]	4	[180,260]	[1,5]
	I10	7	14	5	[30,50]	[4,8]	4	[200,300]	[4,8]
	I11	7	16	4	[50, 80]	[6,10]	5	[260,340]	[6,10]
	I12	7	18	6	[80,120]	[8,12]	5	[280,380]	[8,12]
	I13	8	22	6	[10,30]	[1,5]	5	[200,300]	[1,5]
Cat. C	I14	8	28	6	[30,50]	[4,8]	6	[300,380]	[4,8]
	I15	8	36	8	[50, 80]	[6,10]	6	[340,420]	[6,10]
	I16	9	40	8	[80,120]	[8,12]	8	[400,500]	[8,12]
	I17	9	60	10	[10,30]	[1,5]	8	[200,320]	[1,5]
	I18	10	100	12	[30,50]	[4,8]	10	[280,380]	[4,8]
	I19	12	150	16	[50, 80]	[6,10]	12	[400,560]	[6,10]
	I20	15	200	20	[80,120]	[8,12]	12	[440,600]	[8,12]

* w, c and μ stand for the flow of products, number of servers and service rate in the hubs, respectively

- **GAMLS**: P_{size} = 100; Itr_{max}^{GA} = 100; Selection operator = "Binary tournament selection" (Mohammadi et al., 2015); Crossover & Mutation operators = see Section 4.5.2; E_R = 10%; C_R = 75%; M_R = 15%; $\mathbb{k}$ = 10; Itr_{max}^{ILS} = 80; Local search & Perturbation operations = see Section 4.5.2; Acceptance criterion = 5-10% of gap.
- **NSGA-II**: P_{size} = 200; Itr_{max}^{GA} = 200; Selection operator = "Binary tournament selection"; Crossover & Mutation operators = see Section 4.5.2; E_R = 10%; C_R = 75%; M_R = 15%.
- **HEU1**: P_{size} = 120; Itr_{max}^{ALG1} = 150; $\mathbb{k}$ = 10; Selection operator = based on clusters from $\mathbb{k}$-means; Mutation & Crossover operators = see beginning of Section 5.
- **HEU2**: P_{size} = 120; Itr_{max}^{ALG2} = 150; $\mathbb{k}$ = 10; Selection operator = based on clusters from SOM; Mutation & Crossover operators = see beginning of Section 5.

5.2. Numerical results

This section compares the performance of the proposed GAMLS algorithm with NSGA-II, HEU1 and HEU2 algorithms through generated instances of Table 2. The performance of the algorithms in terms of obtaining (near)-optimal NDP solutions is compared with optimal NDP solutions obtained, over instances of Cat.

A and some of instances of Cat. B, by a well-known augmented ϵ-constraint method (Mohammadi et al., 2019a) programmed using Cplex solver.

The comparison is carried out based on five common metrics used to compare the multi-objective algorithms as (Deb et al., 2000):

- *Convergence Metric* (CM) that evaluates the tightness between the NDP solutions obtained by the algorithms and the optimal NDP solutions (for small- and some of medium-sized instances),
- Quality Metric (QM) that accounts for the number of NDP solutions obtained by an algorithm in comparison to other benchmarking algorithms,
- Divergence Metric (DM) that reports how large the non-dominated frontier of an algorithm is,
- Spacing Metric (SM) that shows that how the NDP solutions have dispersed across the non-dominated frontier, and finally
- Mean Ideal Distance (MID) metric that shows how closed the NDP solutions are from the ideal point (f_1^{min}, f_2^{min}), where f_1^{min} and f_2^{min} are the minimum value of the first and the second objective functions, respectively.

They represent both quantitative and qualitative comparison metric and directly calculated from the NDP solutions. An algorithm with higher values of QM and DM and lower values of CM, SM and MID is a better algorithm.

Tables 3 and 4 report the comparison between the metaheuristic algorithms and the 'Optimal' solutions obtained by the augmented ϵ-constraint method. The reported values for CM, DM, SM and MID metrics are the mean values for problem instances with different sizes. The user CPU-time contains min and max ([min, max]) computational time for solving corresponding instances. Detail results can be found in Appendix A.

According to the mean value of CM in Table 3, it is concluded that the mean tightness of the proposed GAMLS algorithm comparing to the optimal solutions is equal to 1.03% (3% of gap) and 1.11% (11% of gap) for the instances of Cat. A and Cat. B, respectively. For instances of Cat. A and Cat. B, the effectiveness (quality of solutions) of the proposed GAMLS algorithm can be shown in terms of low CM, and its efficiency can be demonstrated by low CPU-time comparing to Cplex solver and other algorithms. Considering the CM for other algorithms from Tables 3 and 4, it is observed that the mean gaps of NSGA-II, HEU1, HEU2 in comparison with the optimal results are, respectively, equal to 11%, 7% and 9% for instances of Cat. A. These values are 32%, 23% and 27% for instances of Cat. B.

Table 3. Comparison with Classical Metaheuristics

Metric	Instance & Algorithm								
	Cat. A			Cat. B			Cat. C		
	Optimal	GAMLS	NSGA-II	Optimal	GAMLS	NSGA-II	Optimal	GAMLS	NSGA-II
CM	-	1.03	1.11	-	1.11	1.32	-	-	-
QM	-	0.76	0.00	-	0.99	0.00	-	1.00	0.00
DM	1.27	1,39	1,14	1.17	1,43	1,01	-	1,47	1,07
SM	0.63	0.37	0.53	0.70	0.32	0.53	-	0.36	0.63
MID	0.71	0.62	1.04	0.66	0.46	0.91	-	0.60	0.97
Time (s)	[124,8976]	[92,177]	[129,219]	[12587,-]	[190,312]	[231,452]	-	[338,752]	[537,1112]

Table 4. Comparison with Learning-based Metaheuristics

Metric	Instance & Algorithm											
	Cat. A				Cat. B				Cat. C			
	Optimal	GAMLS	HEU1	HEU2	Optimal	GAMLS	HEU1	HEU2	Optimal	GAMLS	HEU1	HEU2
CM	-	1.03	1.07	1.09	-	1.11	1.23	1.27	-	-	-	-

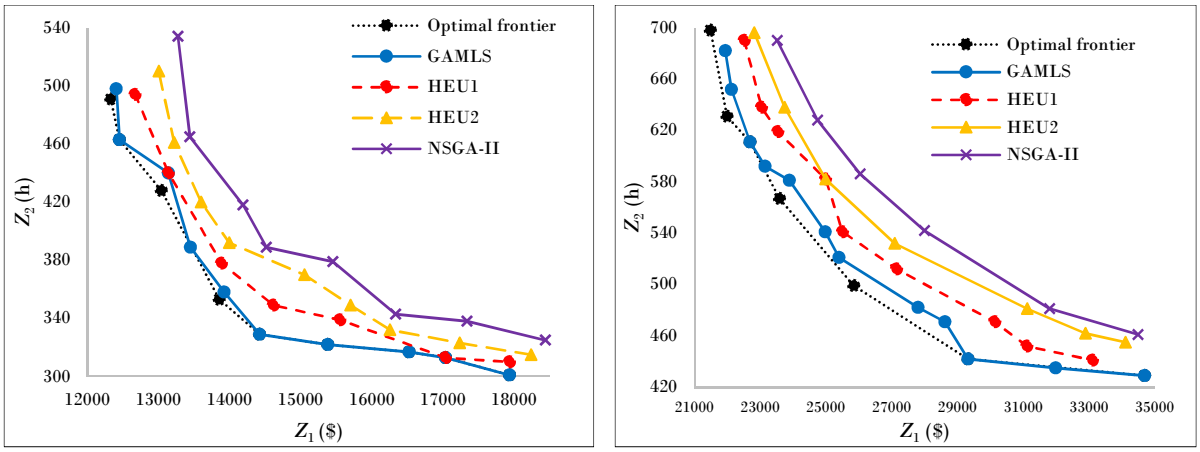
QM	-	0.76	0.34	0.00	-	0.99	0.01	0.00	-	1.00	0.00	0.00
DM	1.27	1.39	1.26	1.22	1.17	1.43	1.30	1.25	-	1.47	1.27	1.18
SM	0.63	0.37	0.37	0.41	0.70	0.32	0.37	0.47	-	0.36	0.36	0.44
MID	0.71	0.62	0.82	0.85	0.66	0.46	0.81	0.94	-	0.60	0.79	0.84
Time (s)	[124, 976]	[92, 177]	[112, 192]	[121, 210]	[12587,-]	[190, 312]	[214, 386]	[227, 439]	-	[338, 752]	[419, 911]	[512, 994]

Although HEU1 and HEU2 use machine learning methods to enhance their performance, the proposed GAMLS algorithm outperforms these algorithms thank to high performance of ILS algorithm in exploiting the neighborhood of NDP solutions. On the other hand, it is also discovered that HEU1 and HEU2 outperforms classical NSGA-II in almost all instances due to their higher performance thank to clustering method and specific search mechanism that keeps a good balance between diversification and intensification. For the values of QM of each algorithm at each category of the problem, please look at both Tables 3 and 4 at the same time since the total share between all four algorithm becomes 100%.

Considering instances of Cat. A and Cat. B, it is observed that the computational time for obtaining optimal NDP by Cplex solver exponentially increases from 124 to 12587 seconds when the size of the instances slightly increases. However, the proposed GAMLS obtains high quality NDP solutions less than 752 seconds even for large-enough instances of Cat. C. The results of Tables 3 and 4 particularly show the superior performance of the proposed GAMLS algorithm in comparison with NSGA-II, HEU1 and HEU2. The outperformance of the proposed GAMLS is supported by lower values of the CM, SM, and MID metric and higher values of DM.

The quality of the proposed GAMLS is further demonstrated by QM, where the NDP solutions of the GAMLS algorithm partially or even completely dominate the NDP solutions obtained by other algorithms in almost all instances. Comparing the performance of the GAMLS algorithm and the NSGA-II reveals that hybridizing the classical NSGA-II with a learning-based ILS significantly improves the performance of the search algorithm in terms of both diversification and intensification.

As another experiment, the learning part of the proposed GAMLS algorithm was removed. Removing the learning part implies that the ILS algorithm be applied on the all the NDP solutions obtained by the NSGA-II. After solving some of the large-sized instances, up to 100 NDP solutions are obtained by NSGA-II. Therefore, the ILS algorithm should be executed 100 times. However, in the proposed GAMLS algorithm, the ILS algorithm is executed only 10 times. On the other hand, we also investigated that removing the learning part does not highly affect the quality of the final non-dominated solutions. Although, higher number of non-dominated solutions were obtained but the majority of the solutions had been already obtained via the GAMLS algorithm. This explanation was added to the revised manuscript without adding the results since they do not provide useful information.



a) Instance I4 b) Instance I8

Figure 5. Optimal vs. metaheuristics: Pareto frontier

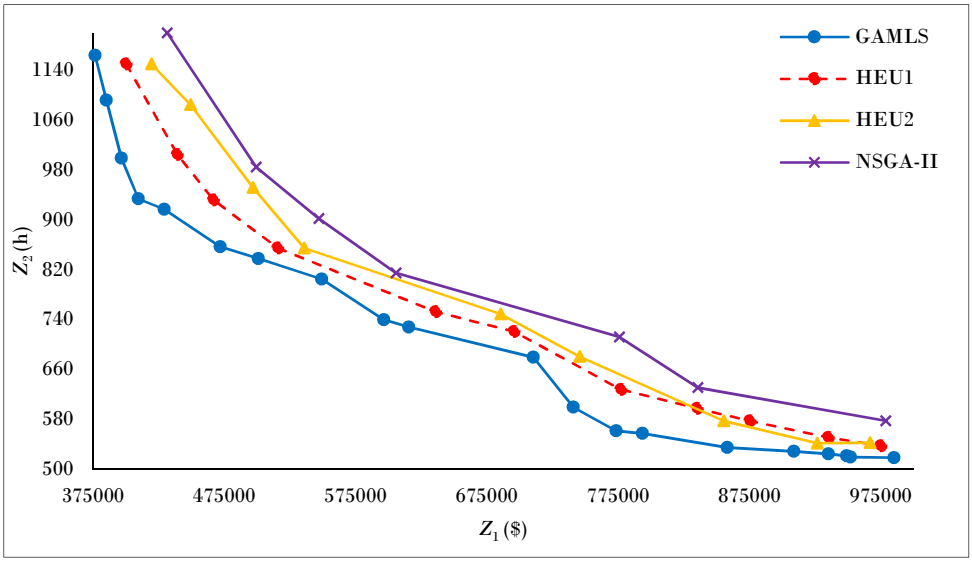


Figure 6. GAMLS vs. NSGA-II, HEU1 & HEU2: Pareto frontier of instance I20

In order to visually show the performance of the proposed GAMLS algorithm, Figures 5a and 5b illustrate the Pareto frontier of the metaheuristic algorithms, respectively for the instances I4 and I8 in comparison with the optimal Pareto frontier obtained by the augmented ϵ-constraint method. In addition, Figure 6 compares the Pareto frontier obtained by the proposed GAMLS and other metaheuristic algorithms for the instance I20. Indeed, we investigated that for the solutions with higher cost, the hubs with higher service level have been located that leads to the lower waiting time in the hubs and consequently lower total transportation time between each pair of OD nodes.

6. Sensitivity Analysis

In this section, the aim is to analyze the sensitivity of the proposed model regarding to the input parameters as well as some other non-parametric factors. The input parameters include flow of products, transportation cost, free-flow transportation time, service rate and number of servers at hubs. For the non-parametric factors, we consider two main sources of variation that affect the performance of the hub network and even the structure of the network. These are 1) Stochastic disruption of the hubs, wherein the hubs are subject to random failures and these failures directly affect the performance of the hub during the process of the products; and 2) Product prioritization, wherein the products are processed based on their priority and the products with

lower priority (i.e., products with lower importance) should wait until the products with higher priority are processed.

Another important issue in any sensitivity analysis, which has been mostly ignored in the literature, is that analyzing the impact of parameters (or non-parametric factor) only on the objective function value could be misleading. Actually, the uncertainty in a parameter is more important if the variation of that parameter affects not only the objective function value, but also the structure of the solution (e.g., the hub-and-spoke network structure in this paper). These types of parameters/factors would be more important to be controlled by the decision makers comparing to those that only affect the objective function values. This sensitivity is then called '*model sensitivity*'. Accordingly, this section investigates both *objective sensitivity* and *model sensitivity* of the input parameters and non-parametric factors. In the following, these sensitivity analyses are performed separately over instance I10.

6.1. Sensitivity to the input parameters

In this section the aim is to investigate the *objective* and *model* sensitivities to the variation of the transportation cost (c_{ij}^p), the free-flow transportation time (t_{ij}^p), the products' flow (w_{ij}^p) and the service rate at hubs (μ_k^p). The sensitivity results are shown based on changes according to the basis scenario (i.e., instance I10). For quantifying the objective sensitivity, the percentage of changes is simply calculated by comparing the objective function values. To quantify the model sensitivity, we calculate a ratio as the number of elements changed in the solution after the parameter variation over the total number of changeable elements in the solution. The elements that construct a solution contains h number of hubs and $|N|\times|P|$ number of connection links from spokes to the hubs. Therefore, the total number of elements in a hub network that may change due to parameter variations is $(|N|\times|P|)+h$. Table 5 shows the objective and model sensitivities to the variation of the transportation cost and free-flow transportation time. In addition, Table 6 shows the sensitivities to the products' flow and hub's service rate. In Tables 5 and 6, column "Sensitivity (%)" represents the mean increase imposed to the corresponding parameters. Negative values in these tables show the reduction in the corresponding values.

From Table 5, it can be observed (as expected) that transportation cost mostly imposes objective sensitivity over first objective function Z_1 that is the sum of the total cost. More interesting, variation of the transportation does not highly affects the model structure such that in the worst case (i.e., 100% increase in the cost), only 8% of the network is changed. In addition, cost increase slightly affects the second objective function Z_2 that is the maximum transportation time between each pair of OD spokes. It can be also seen that increasing the cost up to 15% does not affect the second objective as well as the network structure. It means that the obtained solutions are robust to the variation of the cost up to 15%. Looking at the changes in the free-flow transportation time, inverse results observed comparing to the transportation cost. Changes in the free-flow transportation time highly affects the structure of the network (i.e., high model sensitivity) but has less effect on the first objective function.

It can be then concluded that the second objective function is highly influenced by the structure of the network. Accordingly, companies whose most important objective is the transportation time should be more careful about the variation of those parameters/factors that affect the network structure.

Table 5. Objective & Model sensitivities to input parameters – Part I

Parameter	Change (%)	Sensitivity (%)			Parameter	% of change	Sensitivity (%)		
		Objective increase		Model			Objective increase		Model
		$\mathbb{Z}_1$	$\mathbb{Z}_2$				$\mathbb{Z}_1$	$\mathbb{Z}_2$	
c_{ij}^p	5	4	0	0	t_{ij}^p	5	4	3	2
	10	8	0	0		10	5	9	5
	20	15	0	0		20	8	11	14
	30	22	5	3		30	11	12	19
	40	38	8	4		40	11	16	19
	50	42	12	7		50	19	19	22
	60	53	13	8		60	19	21	22
	70	62	13	8		70	19	23	22
	80	69	13	8		80	23	24	27
	90	79	13	8		90	25	25	29
	100	86	13	8		100	29	25	29

Figures 7 and 8 illustrate how the sensitivity in transportation cost and transportation time affect the objective functions Z_1 and Z_2 as well as the model structure. Comparing Figures 7 and 8 it can be figured out that transportation time has more impact on the objective and model sensitivities.

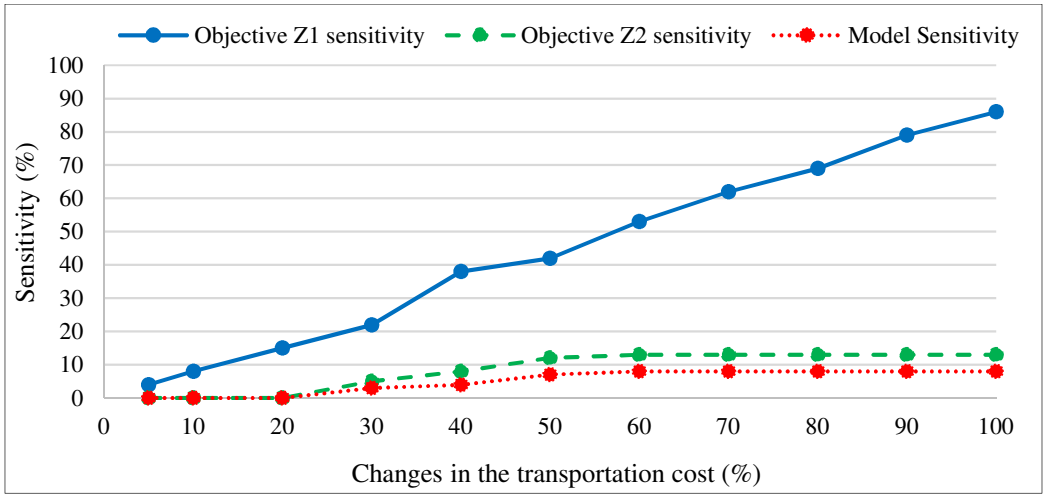


Figure 7. Impact of the transportation cost increase on the objective and model sensitivities

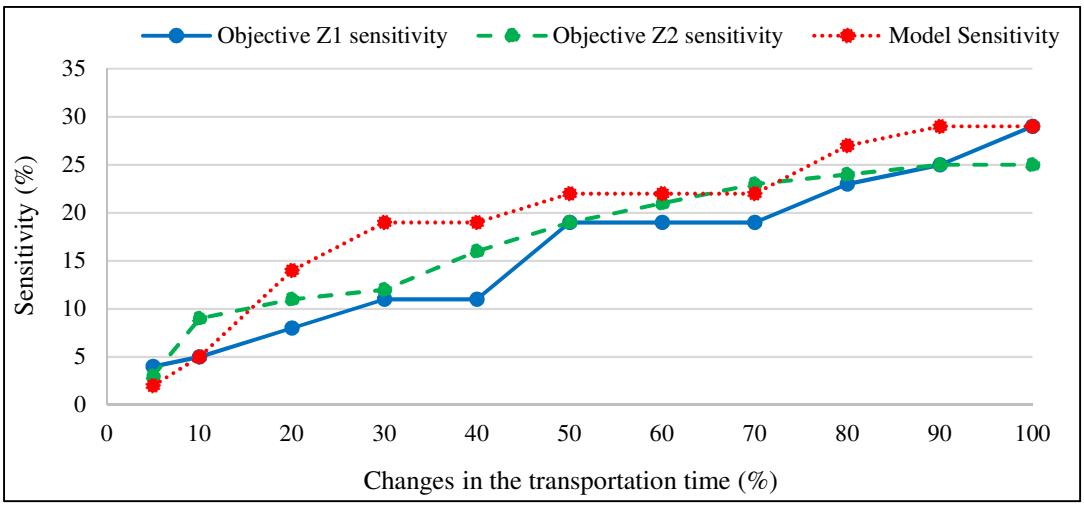


Figure 8. Impact of the transportation time increase on the objective and model sensitivities

From Table 6, more objective and model sensitivities are observed comparing to Table 5. Even small increases in the products' flow highly affects both objective functions as well as the network structure. Once the quantity of the flow increases, the total transportation cost increases somehow linearly. However, the second objective function increases polynomially by the increase of flow. Such significant increase in the second objective function directly relates to the increased congestion of flow in the hubs and the connection links. On the other hand, when the flow increases, the structure of the network is affected significantly. It can be interpreted in such a way that the network tries to adapt itself to the changes in order to absorb these changes as high as possible.

On the other hand, high sensitivities are discovered to the hubs service rate. When the service rate increases, products are processed with higher rate and consequently the waiting time of the products in the hubs is decreased. By accelerating the service rate up to 100%, the transportation time decreases up to 50%. With a simple comparison between the effects of transportation time (t_{ij}^p) and service time (μ_k^p) variations, it is seen that service rate has higher effect on the second objective function and it means that the most of the total transportation time is spent in the hubs and consequently the congestion in the hubs are very important to be taken into account in the design of hub-and-spoke network.

Table 6. Objective & Model sensitivities to input parameters – Part II

Parameter	Change (%)	Sensitivity (%)			Parameter	% of change	Sensitivity (%)		
		Objective increase		Model			Objective increase		Model
		$\mathbb{Z}_1$	$\mathbb{Z}_2$				$\mathbb{Z}_1$	$\mathbb{Z}_2$	
w_{ij}^p	5	3	4	3	μ_k^p	5	0	-2	2
	10	9	11	7		10	0	-4	5
	20	18	19	12		20	2	-10	7
	30	27	24	18		30	3	-12	11
	40	37	32	23		40	-5	-18	15
	50	47	42	25		50	-11	-22	19
	60	56	76	31		60	-11	-29	22
	70	68	105	39		70	-15	-35	24
	80	72	249	42		80	-15	-39	27
	90	83	387	45		90	-15	-42	32
	100	95	551	45		100	-15	-47	35

Figures 9 and 10 illustrate how the sensitivity in transportation cost and transportation time affect the objective functions Z_1 and Z_2 as well as the model structure. Comparing Figures 9 and 10 it can be figured out that transportation time has more impact on the objective and model sensitivities.

These sensitivity analyses figure out that which input parameters are more important to be controlled since their variation in the real setting is unavoidable. Therefore, managers should have higher control on and better estimation of these parameters. In this study, products' flow seems to be the most important parameters that affects the robustness of the solution. On the other hand, service rate of the hubs plays an important role to decrease the second objective function.

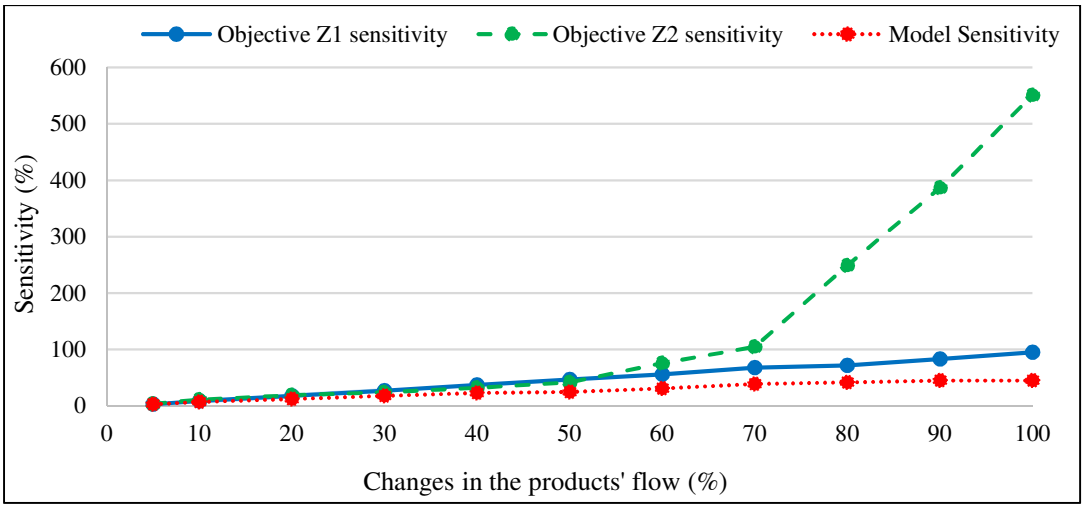


Figure 9. Impact of the products' flow increase on the objective and model sensitivities

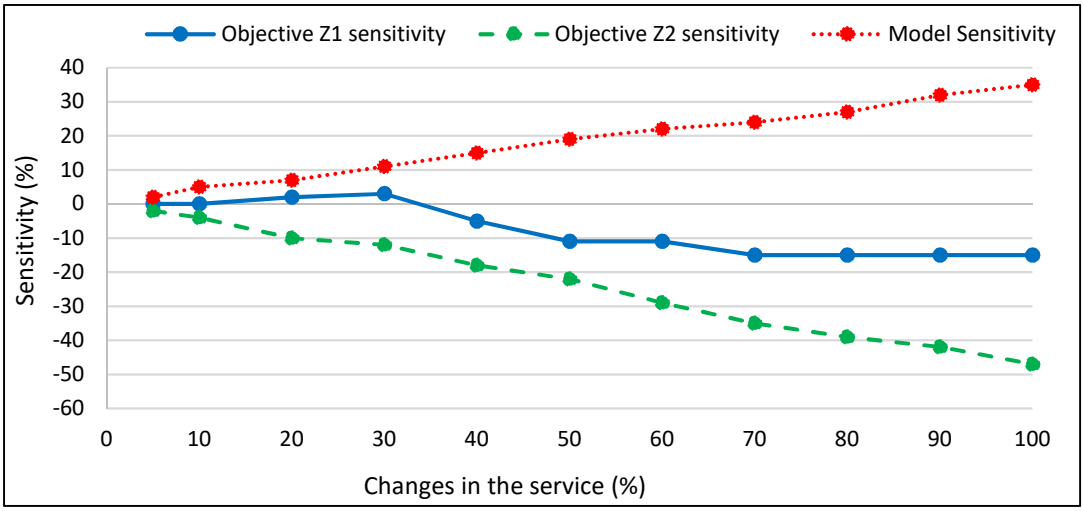


Figure 10. Impact of the products' flow increase on the objective and model sensitivities

6.2. Sensitivity to the stochastic disruption at hubs

We consider that hubs are subject to random failures (Mohammadi and Tavakkoli-Moghaddam, 2016). Accordingly, hub k stochastically fails with rate f_k and consequently the inter-arrival time of failures in hub k is exponentially distributed with mean $1/f_k$. Once a failure happens, the hub becomes completely unavailable and is stochastically retrieved with rate r_k. Accordingly, the time until the hub is retrieved is generally distributed with mean $1/r_k$, standard deviation $\sigma_{R,k}$ and squared coefficient of variation (SCV) of $c_{R,k}^2$ (i.e., $c_{R,k}^2 = r_k\sigma_{R,k}$). Therefore, the mean availability of hub k is $AV_k = \frac{r_k}{r_k+f_k}$ (Morrison and Martin, 2007, Mohammadi et al., 2019b). In addition, the new service rate (μ_k^{p*}) and the new SCV of the service time ($c_{S,pk}^{2*}$) of the hub k are updated as Equations (35) and (36). Equations (12) to (19) are modified accordingly (if necessary).

$$\mu_k^{p*} \equiv AV_k\mu_k^p = \frac{r_k\mu_k^p}{r_k+f_k} \tag{35}$$

$$c_{S,pk}^{2*} = c_{S,pk}^2 + \left(1+c_{S,pk}^2\right)\left(1-\frac{r_k}{r_k+f_k}\right)\frac{\mu_k^p}{(r_k+f_k)} \tag{36}$$

With the same procedure as Section 6.2, the objective and the model sensitivities to the failure rate f_k and retrieve rate r_k are quantified and presented in Table 7. In this experiment, $f_k = 0.2$ and $r_k = 1$.

Based on Equation (35), it is concluded that disruption of the hub services directly affects the hub's service rate that its impact was investigated in Table 6. In Table 7, when the failure rate increases, hubs become less available and consequently the service rate decreases. This decrease leads to the changes in the network structure and consequently the increase of the second objective function. In the other word, once a hub is disrupted, the process of the products is interrupted or done with lower speed; then the congestion is augmented in the hubs. This augmented congestion also increases the waiting time and the value of the second objective function.

On the other hand, increase of the retrieve rate has inverse effect on the augmented congestion. Once the retrieve rate increases, the hub comes back to its designed processing condition and consequently the network structure is not exposed to any changes. Accordingly, managers should take the disruption of the hubs into account as an important non-parametric factor that affects the robustness of the network and try to control the failure rate and/or increase the retrieve rate of the hubs. Figure 11 summarizes the effect of input parameters in the model sensitivity and it can be figured out that through small or big variation, which parameters jeopardize the robustness of the solutions.

Table 7. Objective & Model sensitivities to network disruption

Parameter	Change (%)	Sensitivity (%)			Parameter	% of change	Sensitivity (%)		
		Objective increase		Model			Objective increase		Model
		$\mathbb{Z}_1$	$\mathbb{Z}_2$				$\mathbb{Z}_1$	$\mathbb{Z}_2$	
	5	0	5	4		5	11	15	12
	10	0	6	4		10	10	14	12
	20	2	8	6		20	9	12	10
	30	4	9	7		30	9	12	9
	40	4	11	8		40	8	12	9
f_k	50	5	13	10	r_k	50	8	8	7
	60	7	14	12		60	8	8	6
	70	7	17	12		70	5	5	6
	80	9	18	15		80	3	4	5
	90	10	20	17		90	0	3	4
	100	11	22	19		100	0	3	2

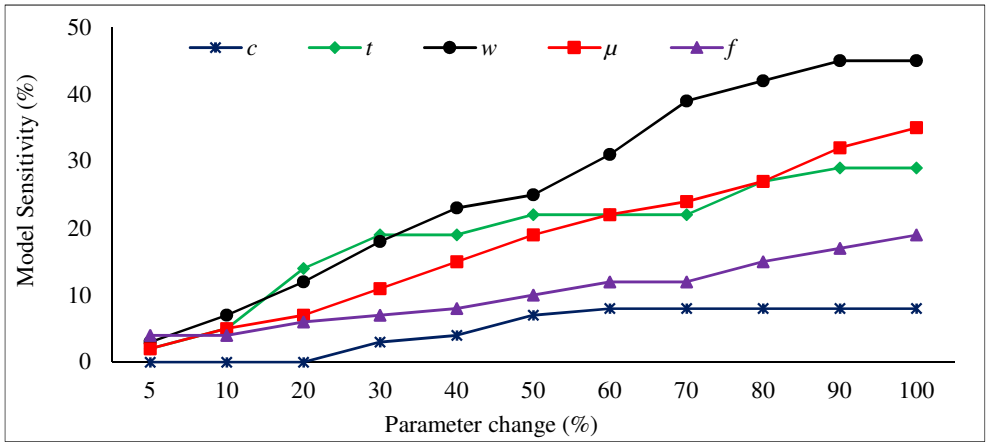


Figure 11. Model sensitivity vs. Parameter changes

6.3. Sensitivity to the products' prioritization

In order to keep competitiveness and the customer satisfaction with significant variety and high variability of their demands, hub-and-spoke system providers usually require to serve the shipment orders

differently, and thus products need to have different priorities. Priorities is usually divided into hot, rush and normal levels (Lee et al., 2008). A product with a higher priority spends shorter waiting time at hubs and should be processed before the lower priority products. In contrary, the products with lower priority need to wait until higher priority products finish their service and the servers become available.

In this paper, we consider the products with multiple priorities in the hub-and-spoke network and figure out the impact product's prioritization on the waiting time of products and particularly the second objective function. For this aim, a priority queuing system is developed for the hub network. All products keep their priority on all visiting hubs. In such a priority queueing system, we assume that an arriving product belongs to a priority class r (r = 1, 2, ..., R). The next product to be served is the customer with the highest priority r (e.g., a product with priority of r has higher priority than a product with priority r' where $r' > r$). The products with the same priority are served upon their arrival to the hub. In addition, no preemption is allowed in the queuing system (i.e., the process of a product already in service is not preempted by an arriving product with higher priority). The mean waiting time for an arriving product p with priority level r at each hub consists of three components as (Bolch et al., 2006): 1) the mean remaining service time at hub k, $\bar{\tau}_k$, of the products in service (if any), 2) the mean service time of products in the queue with the same or higher priority as the tagged product, and 3) the mean service time of products with higher priority that arrive at the queue while the tagged product is in the queue and are served before it.

For simplicity, we consider that each product has a unique priority and several products do not have the same priority (Zahiri et al., 2014; Vahdani and Mohammadi, 2015). Therefore, having P products in the system implies that we have also totally P classes of priority (i.e., $R = P$). We also consider that products are numbered based on their priority. For example, the product with the highest priority is $p = 1$, the product with the second highest priority is $p = 2$, and so on for other products. The mean waiting time of product p (with priority p) in hub k, W_k^p, can be then calculated as Equation (37).

$$W_k^p = \frac{\bar{\tau}_k}{\left(1-\delta_k^p\right)\left(1-\delta_k^{p+1}\right)} \qquad \forall k, p \tag{37}$$

where δ_k^p and $\bar{\tau}_k$ are calculated as Equations (38) and (39).

$$\delta_k^p = \sum_{i=p}^{P} \rho_k^i \qquad \forall k, p \tag{38}$$

$$\bar{\tau}_k \approx \frac{\phi_k^{\mathbb{P}}}{2c_k} \sum_{i=1}^{P} \rho_k^i \frac{c_{A,ik}^2 + c_{S,ik}^2}{\mu_k^p} G_k^{\mathbb{P}} \qquad \forall k \tag{39}$$

This experiment is done on instance I4 containing three products and let's name them as P_1, P_2 and P_3. We consider that P_1 has more priority than P_2 and P_2 has more priority than P_3 and consequently P_1 has higher priority comparing to P_3. In Table 8, we investigate the impact of product prioritization on the product's waiting time at hubs. For this aim, we increases the flow of products simultaneously to see how the increase of flow of the products with high priority affects the waiting time of the products with less priority.

It can be seen from Table 8 that product prioritization in parallel with the flow increase catastrophically affects the second objective function and particularly the waiting time P_2 and P_3. The flow increase of 0% means that the flow of products is kept as the original generated flow of instance I4 and only the prioritization of the product is imposed to the model that leads to decrease in the waiting time of product P_1 but increase in the

waiting time of products P_2 and P_3. These increase and decrease are proportional to the mean waiting time of each products before prioritization. In case of 5%, the same situation happens as the case of 0% but the value of increases are less than the case of 0% because the flow has been increased and this increase itself leads to higher congestion in the hubs and consequently higher mean waiting time in the hubs.

By increasing the flow of P_1 to 80%, the processing of P_3 is blocked since there would be always a unit of P_1 to be processed in a hub. This event happens for P_2 when the flow of P_1 increases up to 100%. On the other hand, the second objective function that attempts to minimize the maximum transportation time between each pair of OD spokes, increases exponentially even to infinity because of the infinite waiting time of P_3. An interesting result is that prioritizing the products lead to decrease of the waiting time of P_1 since it does not need to wait in the hubs in presence of other products. However, it should be noted that prioritizing in parallel with the products' flow augmentation significantly decreases the performance of the hub-and-spoke network in terms of delivery time.

Table 8. Objective & Model sensitivities to flow increase with product's priority

Flow increase (%)	Waiting time increase (%)			Sensitivity (%)		
	P_1	P_2	P_3	Objective increase		Model
				$\mathbb{Z}_1$	$\mathbb{Z}_2$	
0	-20	10	16	3	8	4
5	-17	13	19	6	11	7
10	-10	20	29	11	23	11
20	-3	28	75	14	31	19
30	1	41	142	20	84	21
40	9	89	458	28	324	49
50	14	148	1258	32	895	63
60	28	215	8475	41	6794	74
70	32	428	92187	41	85974	74
80	38	782	Inf	41	Inf	74
90	45	6587	Inf	41	Inf	74
100	49	Inf	Inf	41	Inf	74

7. Conclusion

In a hub-and-spoke network, commodities from different origin spokes are consolidated at hub facilities prior to be routed to an intermediate hub or to be delivered to their final destinations. The aggregation of commodities in the hub facilities allows the exploitation of scale economies due to the utilization of more efficient carriers with higher capacities on hub-to-hub connection links. Despite all advantages of economic scales, this exploitation may lead to commodity overload in a small number of hubs, or even result in heavy-utilization of some hub-to-hub connections. This congestion becomes more critical for transportation companies that employs the hub-and-spoke network for shipment delivery.

This paper addresses the single allocation multi-commodity hub-and-spoke network design under hub congestion. The network is modeled through a bi-objective non-linear mixed integer programming model with congestion in both hubs and hub-to-hub connection links. The proposed bi-objective model minimizes: 1) the total transportation cost and 2) the maximum transportation time between each pair of spokes. A novel aggregation model was developed based on a general GI/G/c queuing system to evaluate the congestion of the flow in the hubs. In addition, a stochastic traffic model was used to account for the congestion of flow traversing the hub-to-hub connection links.

For solving the model, a new hybrid metaheuristic algorithm was developed based on non-dominated sorting genetic algorithm-II (NSGA-II) and a learning-based Iterated Local Search (ILS). A 𝕜-means clustering method is used to link the NSGA-II and ILS to have a powerful search mechanism in terms of both diversification and intensification. The performance of the proposed hybrid algorithm was validated through a benchmark against classical NSGA-II and two new recently developed hybrid algorithms that employ machine learning to improve the performance of the metaheuristic algorithms to show how hybridization of a global search algorithm with a learning-based local search algorithm improves the performance of the searching process. It was observed that adding intelligence of machine learning methods to the metaheuristic algorithm and combining global search and local search algorithms result in high quality solutions even in reasonably low computational time.

Finally, a comprehensive sensitivity analysis was conducted to test not only the sensitivity of the objective functions, but also the robustness of the solutions in terms of their structure. It was concluded that the second objective function is highly influenced by the structure of the network. Accordingly, companies whose most important objective is the transportation time should be more careful about the variation of those parameters/factors (i.e., transportation time, amount of flow and hub's service rate) that affect the network structure.

In addition, the sensitivity of the objective functions and the structure of the solutions was tested to hub disruption and products' priority. Regarding the hub disruption, it was concluded that managers should take the disruption of the hubs into account as an important non-parametric factor that affects the robustness of the network and try to control the failure rate and/or increase the retrieve rate of the hubs. Regarding the products' priority, it was observed that prioritizing of products in parallel with the products' flow augmentation significantly decreases the performance of the hub-and-spoke network in terms of delivery time. Therefore, managers should be careful when prioritizing the products in a network where the flow has the possibility to be augmented.

One of the main further research direction could be studying multiple allocation version of the hub-and-spoke network and investigate if splitting the products and sending them to several hubs can reduce the congestion of the flow in the hubs. Regarding the solution approach, future research effort could be developing new hybrid algorithms that employs machine learning and powerful local search algorithms (e.g., ILS) not only as a posteriori method (e.g., proposed GAMLS algorithm), but also as an interactive way (Sun et al., 2019; Zhang et al., 2016). Such an algorithm is expected to be both intelligent in searching the solution space and powerful in diversification and intensification.

Appendix A

Table A.1. Metaheuristics performance: CM, QM and Time

Instance	Comparison metric												
	CM				QM				Time (s)				
	GAMLS	HEU1	HEU2	NSGA-II	GAMLS	HEU1	HEU2	NSGA-II	Optimal	GAMLS	HEU1	HEU2	NSGA-II
I1	1.00	1.00	1.00	1.00	0.50	0.50	0.00	0.00	124	92	112	121	129
I2	1.00	1.00	1.00	1.02	0.60	0.40	0.00	0.00	295	111	119	129	136
I3	1.00	1.04	1.05	1.09	0.80	0.20	0.00	0.00	484	125	125	141	152
I4	1.03	1.09	1.11	1.12	0.80	0.20	0.00	0.00	1286	132	142	166	174
I5	1.05	1.12	1.14	1.18	0.90	0.10	0.00	0.00	3485	154	183	193	198
I6	1.09	1.17	1.19	1.22	1.00	0.00	0.00	0.00	8976	177	192	210	219
I7	1.10	1.20	1.24	1.29	1.00	0.00	0.00	0.00	12587	190	214	227	231
I8	1.13	1.25	1.30	1.34	1.00	0.00	0.00	0.00	18654	219	234	239	242
I9	-	-	-	-	1.00	0.00	0.00	0.00	> 10h	228	261	279	289
I10	-	-	-	-	1.00	0.00	0.00	0.00	-	247	289	317	329
I11	-	-	-	-	1.00	0.00	0.00	0.00	-	273	316	359	364
I12	-	-	-	-	1.00	0.00	0.00	0.00	-	294	338	412	423
I13	-	-	-	-	1.00	0.00	0.00	0.00	-	312	386	439	452
I14	-	-	-	-	1.00	0.00	0.00	0.00	-	338	419	512	537
I15	-	-	-	-	1.00	0.00	0.00	0.00	-	384	459	549	589
I16	-	-	-	-	1.00	0.00	0.00	0.00	-	398	483	589	612
I17	-	-	-	-	1.00	0.00	0.00	0.00	-	419	528	629	699
I18	-	-	-	-	1.00	0.00	0.00	0.00	-	556	642	732	786
I19	-	-	-	-	1.00	0.00	0.00	0.00	-	649	829	896	923
I20	-	-	-	-	1.00	0.00	0.00	0.00	-	752	911	994	1112

Table A.2. Optimal vs. GAMLS & NSGA-II: DM, SM and MID

Instance	Comparison metric								
	DM			SM			MID		
	Optimal	GAMLS	NSGA-II	Optimal	GAMLS	NSGA-II	Optimal	GAMLS	NSGA-II
I1	1.41	1.52	1.19	0.54	0.44	0.61	0.72	0.59	1.05
I2	1.22	1.35	1.29	0.64	0.27	0.53	0.71	0.68	0.82
I3	1.09	1.25	1.20	0.72	0.41	0.52	0.81	0.61	1.19
I4	1.26	1.29	1.09	0.59	0.24	0.45	0.67	0.67	1.18
I5	1.23	1.45	1.13	0.78	0.35	0.62	0.62	0.49	0.86
I6	1.41	1.47	0.95	0.53	0.48	0.46	0.73	0.67	1.11
I7	1.03	1.51	1.08	0.67	0.38	0.45	0.68	0.48	0.61
I8	1.31	1.48	1.13	0.73	0.24	0.48	0.63	0.47	0.93
I9	-	1.34	1.25	-	0.48	0.46	-	0.45	0.78
I10	-	1.51	0.91	-	0.23	0.62	-	0.42	1.12
I11	-	1.14	0.86	-	0.36	0.64	-	0.44	0.85
I12	-	1.52	0.89	-	0.33	0.54	-	0.43	0.92
I13	-	1.51	0.84	-	0.23	0.51	-	0.51	1.19
I14	-	1.42	1.15	-	0.33	0.61	-	0.47	0.64
I15	-	1.48	1.16	-	0.38	0.67	-	0.51	1.04
I16	-	1.58	1.28	-	0.36	0.65	-	0.69	1.15
I17	-	1.55	0.88	-	0.41	0.46	-	0.68	1.02
I18	-	1.43	1.03	-	0.39	0.77	-	0.56	1.09
I19	-	1.47	0.86	-	0.29	0.67	-	0.61	0.89
I20	-	1.33	1.19	-	0.33	0.58	-	0.70	0.99

Table A.3. GAMLS vs. HEU1 & HEU2: DM, SM and MID

Instance	Comparison metric								
	DM			SM			MID		
	GAMLS	HEU1	HEU2	GAMLS	HEU1	HEU2	GAMLS	HEU1	HEU2
I1	1.52	1.24	1.22	0.44	0.45	0.43	0.59	0.98	1.02
I2	1.35	1.23	1.11	0.27	0.37	0.36	0.68	0.56	0.93
I3	1.25	1.25	1.15	0.41	0.41	0.42	0.61	0.86	0.67
I4	1.29	1.25	1.17	0.24	0.32	0.45	0.67	0.96	0.73
I5	1.45	1.37	1.29	0.35	0.31	0.38	0.49	0.88	0.87
I6	1.47	1.24	1.35	0.48	0.34	0.39	0.67	0.65	0.89
I7	1.51	1.35	1.32	0.38	0.39	0.48	0.48	0.94	1.08
I8	1.48	1.22	1.19	0.24	0.33	0.44	0.47	0.78	0.97
I9	1.34	1.28	1.23	0.48	0.27	0.45	0.45	0.98	1.06
I10	1.51	1.22	1.18	0.23	0.44	0.42	0.42	0.79	1.03
I11	1.14	1.36	1.32	0.36	0.34	0.49	0.44	0.59	1.07
I12	1.52	1.33	1.31	0.33	0.38	0.54	0.43	0.66	0.69
I13	1.51	1.36	1.23	0.23	0.41	0.48	0.51	0.94	0.71
I14	1.42	1.26	1.23	0.33	0.25	0.41	0.47	0.56	0.94
I15	1.48	1.12	1.05	0.38	0.37	0.57	0.51	0.85	0.86
I16	1.58	1.28	1.14	0.36	0.45	0.49	0.69	0.78	0.71
I17	1.55	1.34	1.25	0.41	0.43	0.45	0.68	0.92	1.05
I18	1.43	1.42	1.24	0.39	0.34	0.45	0.56	0.87	0.67
I19	1.47	1.15	1.11	0.29	0.31	0.37	0.61	0.61	0.83
I20	1.33	1.29	1.27	0.33	0.35	0.36	0.70	0.96	0.85

References

Alkaabneh, F., Diabat, A., & Elhedhli, S. (2019). A Lagrangian heuristic and GRASP for the hub-and-spoke network system with economies-of-scale and congestion. Transportation Research Part C: Emerging Technologies, 102, 249-273.

Alumur, S., Kara, B.Y., 2008. Network hub location problems: The state of the art. Eur. J. Oper. Res. 190, 1–21. https://doi.org/10.1016/j.ejor.2007.06.008

Azizi, N., Vidyarthi, N., & Chauhan, S. S. (2018). Modelling and analysis of hub-and-spoke networks under stochastic demand and congestion. Annals of Operations Research, 264(1-2), 1-40.

Azizmohammadi, R., Amiri, M., Tavakkoli-Moghaddam, R., Mohammadi, M., 2013. Solving a Redundancy Allocation Problem by a Hybrid Multi-objective Imperialist Competitive Algorithm. Int. J. Eng. - Trans. C Asp. 26, 1031–1042.

Bolch, G., Greiner, S., Meer, H. de, Trivedi, K.S., 2006. Queueing Networks and Markov Chains: Modeling and Performance Evaluation with Computer Science Applications. John Wiley & Sons.

Calvet, L., de Armas, J., Masip, D., & Juan, A. A. (2017). Learnheuristics: hybridizing metaheuristics with machine learning for optimization with dynamic inputs. Open Mathematics, 15(1), 261-280.

Chiang, T.-C., Cheng, H.-C., Fu, L.-C., 2011. NNMA: An effective memetic algorithm for solving multiobjective permutation flow shop scheduling problems. Expert Syst. Appl. 38, 5986–5999. https://doi.org/10.1016/j.eswa.2010.11.022

Correia, I., Nickel, S., & Saldanha-da-Gama, F., 2018. A stochastic multi-period capacitated multiple allocation hub location problem: Formulation and inequalities. *Omega*, *74*, 122-134.

da Graça Costa, M., Captivo, M.E., Clímaco, J., 2008. Capacitated single allocation hub location problem—A bi-criteria approach. Comput. Oper. Res., Part Special Issue: Topics in Real-time Supply Chain Management 35, 3671–3695. https://doi.org/10.1016/j.cor.2007.04.005

de Camargo, R.S., Miranda, G., 2012. Single allocation hub location problem under congestion: Network owner and user perspectives. Expert Syst. Appl. 39, 3385–3391. https://doi.org/10.1016/j.eswa.2011.09.026

de Camargo, R.S., Miranda Jr., G., Ferreira, R.P.M., Luna, H.P., 2009. Multiple allocation hub-and-spoke network design under hub congestion. Comput. Oper. Res., New developments on hub location 36, 3097–3106. https://doi.org/10.1016/j.cor.2008.10.004

Deb, K., Agrawal, S., Pratap, A., Meyarivan, T., 2000. A Fast Elitist Non-dominated Sorting Genetic Algorithm for Multi-objective Optimization: NSGA-II, in: Schoenauer, M., Deb, K., Rudolph, G., Yao, X., Lutton, E., Merelo, J.J., Schwefel, H.-P. (Eds.), Parallel Problem Solving from Nature PPSN VI, Lecture Notes in Computer Science. Springer Berlin Heidelberg, pp. 849–858.

Dukkanci, O., Peker, M., & Kara, B. Y. (2019). Green hub location problem. Transportation Research Part E: Logistics and Transportation Review, 125, 116-139.

Elhedhli, S., Hu, F.X., 2005. Hub-and-spoke network design with congestion. Comput. Oper. Res. 32, 1615–1632. https://doi.org/10.1016/j.cor.2003.11.016

Elhedhli, S., Wu, H., 2009. A Lagrangean Heuristic for Hub-and-Spoke System Design with Capacity Selection and Congestion. Inf. J. Comput. 22, 282–296. https://doi.org/10.1287/ijoc.1090.0335

Grove, P.G., O'Kelly, M.E., 1986. Hub Networks and Simulated Schedule Delay. Pap. Reg. Sci. 59, 103–119. https://doi.org/10.1111/j.1435-5597.1986.tb00985.x

Hu, L., Zhu, J. X., Wang, Y., & Lee, L. H. (2018). Joint design of fleet size, hub locations, and hub capacities for third-party logistics networks with road congestion constraints. Transportation Research Part E: Logistics and Transportation Review, 118, 568-588.

Ishfaq, R., & Sox, C. R., 2012. Design of intermodal logistics networks with hub delays. European Journal of Operational Research, 220(3), 629-641.

Jourdan, L., Dhaenens, C., Talbi, E.-G., 2006. Using Datamining Techniques to Help Metaheuristics: A Short Survey, in: Almeida, F., Blesa Aguilera, M.J., Blum, C., Moreno Vega, J.M., Pérez Pérez, M., Roli, A., Sampels, M. (Eds.), Hybrid Metaheuristics, Lecture Notes in Computer Science. Springer Berlin Heidelberg, pp. 57–69.

Karimi-Mamaghan, M., Mohammadi, M., Jula, P., Pirayesh, A., & Ahmadi, H. (2020). A Hybrid Metaheuristic for a Multi-objective Agile Inspection Planning Model under Uncertainty. European Journal of Operational Research. In press. DOI: 10.1016/j.ejor.2020.01.061.

Kian, R., & Kargar, K. (2016). Comparison of the formulations for a hub-and-spoke network design problem under congestion. Computers & Industrial Engineering, 101, 504-512.

Kleinrock, L., 2007. Communication Nets: Stochastic Message Flow and Delay. Courier Corporation.

Lee, A.H.I., Huang, T.-H., Kang, H.-Y., 2008. A priority mix planning model for semiconductor fabrication under an uncertain information environment. J. Inf. Optim. Sci. 29, 377–400. https://doi.org/10.1080/02522667.2008.10699811

Likas, A., Vlassis, N., J. Verbeek, J., 2003. The global k-means clustering algorithm. Pattern Recognit., Biometrics 36, 451–461. https://doi.org/10.1016/S0031-3203(02)00060-2

Lo, H.K., Tung, Y.-K., 2003. Network with degradable links: capacity analysis and design. Transp. Res. Part B Methodol. 37, 345–363. https://doi.org/10.1016/S0191-2615(02)00017-6

Marianov, V., Serra, D., 2003. Location models for airline hubs behaving as M/D/c queues. Comput. Oper. Res. 30, 983–1003. https://doi.org/10.1016/S0305-0548(02)00052-7

Mohammadi, M., Dauzère-Pérès, S., Yugma, C., 2018. Performance evaluation of single and multi-class production systems using an approximating queuing network. Int. J. Prod. Res. 0, 1–27. https://doi.org/10.1080/00207543.2018.1492163

Mohammadi, M., Jolai, F., Rostami, H., 2011. An M/M/c queue model for hub covering location problem. Math. Comput. Model. 54, 2623–2638. https://doi.org/10.1016/j.mcm.2011.06.038

Mohammadi, M., Jolai, F., Tavakkoli-Moghaddam, R., 2013. Solving a new stochastic multi-mode p-hub covering location problem considering risk by a novel multi-objective algorithm. Appl. Math. Model. 37, 10053–10073. https://doi.org/10.1016/j.apm.2013.05.063

Mohammadi, M., Jula, P., Tavakkoli-Moghaddam, R., 2019a. Reliable single-allocation hub location problem with disruptions. Transp. Res. Part E Logist. Transp. Rev. 123, 90–120. https://doi.org/10.1016/j.tre.2019.01.008

Mohammadi, M., Dauzère-pérès, S., Yugma, C., Karimi-Mamaghan, M., 2019b. A queue-based aggregation approach for performance evaluation of a production system with an AMHS. Computers & Operations Research, 2019, doi: 10.1016/j.cor.2019.104838.

Mohammadi, M., Jula, P., Tavakkoli-Moghaddam, R., 2017. Design of a reliable multi-modal multi-commodity model for hazardous materials transportation under uncertainty. Eur. J. Oper. Res. 257, 792–809. https://doi.org/10.1016/j.ejor.2016.07.054

Mohammadi, M., Siadat, A., Dantan, J.-Y., Tavakkoli-Moghaddam, R., 2015. Mathematical modelling of a robust inspection process plan: Taguchi and Monte Carlo methods. Int. J. Prod. Res. 53, 2202–2224. https://doi.org/10.1080/00207543.2014.980460

Mohammadi, M., Tavakkoli-Moghaddam, R., 2016. Design of a fuzzy bi-objective reliable p-hub center problem. J. Intell. Fuzzy Syst. 30, 2563–2580. https://doi.org/10.3233/IFS-151846

Mohammadi, M., Tavakkoli-Moghaddam, R., Siadat, A., Rahimi, Y., 2016. A game-based meta-heuristic for a fuzzy bi-objective reliable hub location problem. Eng. Appl. Artif. Intell. 50, 1–19. https://doi.org/10.1016/j.engappai.2015.12.009

Mohammadi, M., Torabi, S.A., Tavakkoli-Moghaddam, R., 2014. Sustainable hub location under mixed uncertainty. Transp. Res. Part E Logist. Transp. Rev. 62, 89–115. https://doi.org/10.1016/j.tre.2013.12.005

Morrison, J. R., & Martin, D.P., 2007. Practical extensions to cycle time approximations for the g/g/m queue with applications. IEEE Transactions on Automation Science and Engineering, 4(4), 523-532.

Oliveto, P. S., & Witt, C. (2015). Improved time complexity analysis of the simple genetic algorithm. Theoretical Computer Science, 605, 21-41.

Özgün-Kibiroğlu, Ç., Serarslan, M. N., & Topcu, Y. İ. (2019). Particle swarm optimization for uncapacitated multiple allocation hub location problem under congestion. Expert Systems with Applications, 119, 1-19.

Rahimi, Y., Tavakkoli-Moghaddam, R., Mohammadi, M., Sadeghi, M., 2016. Multi-objective hub network design under uncertainty considering congestion: An M/M/c/K queue system. Appl. Math. Model. 40, 4179–4198. https://doi.org/10.1016/j.apm.2015.11.019

Rodríguez, V., Alvarez, M.J., Barcos, L., 2007. Hub location under capacity constraints. Transp. Res. Part E Logist. Transp. Rev. 43, 495–505. https://doi.org/10.1016/j.tre.2006.01.005

Satyam, K., Krishnamurthy, A., 2008. Performance evaluation of a multi-product system under CONWIP control. IIE Trans. 40, 252–264. https://doi.org/10.1080/07408170701488086

Sedehzadeh, S., Tavakkoli-Moghaddam, R., Mohammadi, M., & Jolai, F., 2014. Solving a new priority m/m/c queue model for a multimode hub covering location problem by multi-objective parallel simulated annealing. Economic Computation & Economic Cybernetics Studies & Research, 48(4).

Sun, J., Zhang, H., Zhou, A., Zhang, Q., Zhang, K., 2019. A new learning-based adaptive multi-objective evolutionary algorithm. Swarm Evol. Comput. 44, 304–319. https://doi.org/10.1016/j.swevo.2018.04.009

Taleizadeh, Ata Allah, Maryam Karimi Mamaghan, and Seyed Ali Torabi, 2018. A possibilistic closed-loop supply chain: pricing, advertising and remanufacturing optimization. Neural Computing and Applications: 1-21.

Vahdani, B., Mohammadi, M., 2015. A bi-objective interval-stochastic robust optimization model for designing closed loop supply chain network with multi-priority queuing system. Int. J. Prod. Econ. 170, 67–87. https://doi.org/10.1016/j.ijpe.2015.08.020

Yang, T. H., & Chiu, T. Y. (2016). Airline hub-and-spoke system design under stochastic demand and hub congestion. Journal of Industrial and Production Engineering, 33(2), 69-76.

Wang, X., Tang, L., 2017. A machine-learning based memetic algorithm for the multi-objective permutation flowshop scheduling problem. Comput. Oper. Res. 79, 60–77. https://doi.org/10.1016/j.cor.2016.10.003

Zahiri, B., Tavakkoli-Moghaddam, R., Mohammadi, M., Jula, P., 2014. Multi-objective design of an organ transplant network under uncertainty. Transp. Res. Part E Logist. Transp. Rev. 72, 101–124. https://doi.org/10.1016/j.tre.2014.09.007

Zhalechian, M., Torabi, S.A., Mohammadi, M., 2018. Hub-and-spoke network design under operational and disruption risks. Transp. Res. Part E Logist. Transp. Rev. 109, 20–43. https://doi.org/10.1016/j.tre.2017.11.001

Zhang, J., Zhan, Z., Lin, Y., Chen, N., Gong, Y., Zhong, J., Chung, H.S.H., Li, Y., Shi, Y., 2011. Evolutionary Computation Meets Machine Learning: A Survey. IEEE Comput. Intell. Mag. 6, 68–75. https://doi.org/10.1109/MCI.2011.942584

Zhang, H., Zhou, A., Song, S., Zhang, Q., Gao, X.-Z., Zhang, J., 2016. A Self-Organizing Multiobjective Evolutionary Algorithm. IEEE Trans. Evol. Comput. 20, 792–806. https://doi.org/10.1109/TEVC.2016.2521868

Zheng, J., Zhang, W., Qi, J., & Wang, S. (2019). Canal effects on a liner hub location problem. Transportation Research Part E: Logistics and Transportation Review, 130, 230-247.