Incremental elicitation of the criteria weights of SRMP using a regret-based query selection strategy

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Arwa Khannoussi [1], **Nawal Benabbou** [2], **Alexandru-Liviu Olteanu** [3] and **Patrick Meyer** [4]

Abstract. The Simple Ranking Method using Reference Profiles (or SRMP) is a Multi-Criteria Decision Aiding (MCDA) technique based on the outranking paradigm, which allows to rank decision alternatives according to the preferences of a decision maker (DM). Inferring the preference parameters of such a model can lead to a cognitive fatigue of the DM, who is often asked to express several preferential statements about pairs of alternatives during the elicitation process. To limit the DM's effort, an incremental elicitation process can be used to select informative pairs of alternatives to be presented to the DM sequentially with the aim of refining the SRMP model until a robust recommendation can be made. In this paper, we propose a heuristic for selecting the pairs of alternatives submitted sequentially to the DM for evaluation, using a regret-based elicitation approach. We restrict our proposal to the elicitation of a subset of parameters of the SRMP model, namely the criteria weights, and show that in this context, the proposed heuristic outperforms previously studied query selection criteria.

1 Introduction

Multi-Criteria Decision Aiding (MCDA) [22] is a methodology used to support decision makers (DMs) when multiple criteria have to be taken into consideration, whether the goal is to *choose* among a set of decision alternatives, *sort* them into predefined categories, or *rank* them from the "best" to the "worst" one. A large variety of MCDA techniques have been proposed to help the DM solve complex decision problems by taking into account his/her preferences and are roughly classified into three approaches: [5] (i) Multi-Attribute Value Theory (MAVT) [12] , (ii) outranking-based approaches [9] and (iii) rule-based models [11]. In this paper, we focus on the Simple Ranking Method using Reference Profiles (SRMP) [4, 20], which is based on the outranking paradigm. This ranking method is very useful in various real-world applications as it can easily handle heterogeneous evaluation scales, while at the same time constructing a transitive global weak preference relation. Its similarity to normative outranking-based sorting approaches also helps to justify the decision recommendations [6].

Whatever the type of the method, the preferences of the DM have to be elicited (learned) in order to be able to apply it. An indirect elicitation approach is usually employed, in which the DM is only asked to express holistic judgments on alternatives (e.g., assignment examples, a partial pre-order on the alternatives, pairwise comparisons of alternatives). In the literature, this indirect approach is divided into two categories [17]: *"batch elicitation"*, in which the learning data is given all at once to the learning algorithm [5], and *"incremental elicitation"*, in which the learning data arrives sequentially and the model is improved iteratively [3, 10, 7]. In this paper, we study the potential of incremental elicitation to learn some of the preference parameters of the SRMP model, namely the criteria weights.

In the *batch* setting, some recent works propose elicitation approaches for the SRMP model, in which the DM is asked to express his/her preference through pairwise comparisons of alternatives that are used to infer the model's parameters (e.g. criteria weights, profiles and their lexicographic order). In [18], the preference parameters are inferred by solving a mixed integer linear optimization problem (MIP) which is defined using binary comparisons (preference/indifference) of alternatives provided by the DM. Next to that, Belahcène et al. [1] propose another approach which consists in solving a Boolean satisfiability (SAT) problem. Compared to the previous proposition, this approach is faster and can handle larger sets of pairwise comparisons of alternatives. Liu et al. [15] propose a metaheuristic to elicit the parameters of an SRMP model which is faster than the MIP approach but it does not guarantee to find the *best model* which would perfectly match the input pairwise comparisons.

Next to that, Khannoussi et al. [14] propose to learn the parameters of the SRMP model via an *incremental* elicitation process to decrease the cognitive fatigue of the DM by reducing the number of learning examples (pairwise comparisons of alternatives) that are necessary to make a good recommendation. At every step of the incremental elicitation process, a pair of alternatives is selected using a query selection heuristic and presented to the DM in order to improve the current SRMP model through a learning algorithm. The proposed heuristic (called $\mathcal{H}_{mp}$ hereafter) consists in selecting a pair of alternatives that involves a maximal number of profiles in their comparison through the current SRMP model. In this paper, we propose a new query selection heuristic for the SRMP model which is inspired by regret-based incremental elicitation approaches studied in the context of MAVT.

In MAVT methods, it has been proposed to use the *minimax regret decision criterion* within an incremental elicitation approach in order to make robust decisions under preference imprecision and generate informative preference queries [23, 3]. The idea is to save preference queries by identifying the part of preference information that is necessary to solve the instance under consideration without seeking to precisely specify the decision model. This approach, sometimes referred to as *regret-based incremental elicitation*, was efficiently used in various decision contexts, such as multicriteria decision making [2], collective decision making [16], and decision making under risk [19].

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The adaptation of minimax regret elicitation strategies to outranking methods is not straightforward as regrets are usually defined using direct comparisons of solutions (without considering any reference profile). In this paper, we propose a new definition of regrets which takes into account the reference profiles of the SRMP model and that enables to generate informative preference queries during the incremental elicitation process.

The rest of this article is structured in the following way: First, we recall some useful background on the SRMP model in Section 2. Then, in Section 3 we introduce a new notion of regret for the SRMP model together with a regret-based query selection heuristic which efficiently selects the pairs of alternatives to be presented to the DM at every step of the incremental elicitation procedure. We also propose a mixed-integer linear programming formulation to solve the corresponding regret-based optimization problems at every step of the procedure. Finally, we report the results of numerical tests on generated data in Section 4, showing that our procedure is more efficient than other heuristics proposed in the literature.

2 The SRMP model

In outranking methods, an "at least as good as" relation is built between alternatives evaluated on multiple criteria. This binary relation, called "outranking relation" [21] is often denoted by $\succsim$. An alternative a outranks another one, say b, denoted by $a \succsim b$, if there are strong enough arguments to declare that a is at least as good as b and if there is no essential reason to refute that statement. However, comparing all alternatives pairwisely according to such a relation does not necessarily generate a transitive relation, and may result in cycles in the relation, thus making it impossible to create a complete ranking [8]. To avoid this problem, one can make use of a so-called *reference point* when comparing two alternatives [20]. The basic idea is that a is said to be preferred to be b if and only if the outranking relation between a and the reference point is "stronger" than the outranking relation between b and the reference point. In this paper, we focus on the SRMP outranking method which makes use of several reference points, their order, as well as criteria weights to define the "strength" of those arguments. Let us now show how this method is implemented more formally.

Let $\mathcal{A}$ denote a set of n alternatives that are evaluated with respect to a set of m criteria denoted by $M = \{1, \ldots, m\}$. The evaluation of an alternative $a \in \mathcal{A}$ on criterion $j \in M$ is denoted by a_j so that a can be identified with its performance vector $a \equiv (a_1, \ldots, a_m)$. For every criterion $j \in M$, we are given a preorder denoted by $\succsim_j$ such that $a_j \succsim_j b_j$ whenever a is at least as good as b on criterion j. SRMP is defined by several parameters, whose values may differ from one DM to another:

- $\mathcal{P}$: a set of k reference profiles denoted by $p^h \equiv (p_1^h, \ldots, p_m^h)$, with $h \in \{1, \ldots, k\}$, that dominate each other, i.e. $p_j^h \succsim_j p_j^{h+1}$ for all $h \in \{1, \ldots, k-1\}$.
- σ: a permutation of $\{1, \ldots, k\}$ which represents the order in which the profiles will be used when comparing alternatives, defining thus a lexicographic order.
- $w \equiv (w_1, \ldots, w_m)$: a vector of weights attached to criteria such that $w_j > 0$ for all $j \in M$ and $\sum\limits_{j \in M} w_j = 1$.

For any alternative $a \in \mathcal{A}$ and any profile $p^h \in \mathcal{P}$, we define the weight $f_w(a, p^h)$ representing the strength of arguments supporting the statement "a is at least as good as p^h" as follows:

$$f_w(a, p^h) = \sum_{j \in C(a, p^h)} w_j \tag{1}$$

where $C(a, p^h) = \{j \in M : a_j \succsim_j p_j^h\}$ is set of criteria on which alternative a is at least as good as profile p^h. When comparing two alternatives $a, b \in \mathcal{A}$ through a profile $p^h \in \mathcal{P}$, three cases can be distinguished:

- if $f_w(a, p^h) > f_w(b, p^h)$, then a is said to be strictly preferred to b with respect to p^h, which is denoted by $a \succ_{p^h} b$.
- if $f_w(a, p^h) = f_w(b, p^h)$, then a is said to be indifferent to b with respect to p^h, which is denoted by $a \sim_{p^h} b$.
- if $f_w(a, p^h) < f_w(b, p^h)$, then b is said to be strictly preferred to a with respect to p^h, which is denoted by $b \succ_{p^h} a$.

The outranking relation $\succsim$ representing the DM's preferences is defined by sequentially considering the profiles $p^{\sigma(1)}, p^{\sigma(2)}, \ldots, p^{\sigma(k)}$ according to the lexicographic order. Intuitively, a preference between a and b is formed as soon as we encounter a profile in the lexicographic order for which a is preferred to b or vice-versa. Otherwise, a and b are considered as indifferent, which means that no profile has been able to discriminate between them. More formally, for any two alternatives $a, b \in \mathcal{A}$, we say that:

- a is strictly preferred to b, denoted by $a \succ b$, if and only if:

$$\exists h \in \{1, \ldots, k\} \text{ s.t. } a \succ_{p^{\sigma(h)}} b \text{ and } \forall l < h,\ a \sim_{p^{\sigma(l)}} b \tag{2}$$

- a is indifferent to b, denoted by $a \sim b$, if and only if:

$$\forall h \in \{1, \ldots, k\}, a \sim_{p^{\sigma(h)}} b \tag{3}$$

3 A Regret-Based Incremental Elicitation Process

The SRMP method presented in the previous section involves a weighting vector $w \equiv (w_1, \ldots, w_j)$ representing the importance of criteria according to the DM's preferences. In this paper, we assume that this parameter w is initially not known, and our aim is to propose an incremental elicitation procedure for its assessment. The remaining preferential parameters are supposed to be known.

Similarly to [14], at each iteration step of the incremental elicitation process, a heuristic is used to select a pair of alternatives that will be presented to the DM. He/she then expresses his/her preferences on these alternatives, either in the form of a strict preference or an indifference. This information is then added to the set of preference statements obtained so far, which is then used to restrict the set of admissible parameters for the SRMP model. The process stops after a certain number of iterations, or when the DM considers that the SRMP model, that can be constructed at any point in the process, is faithful to his/her preferences. We propose here a new query selection heuristic that is inspired by the minimax regret decision criterion which is commonly used in the MAVT setting. Our heuristic helps in reducing the number of preference queries that are needed for converging towards a good enough model in practice, and allows to stop the elicitation process sooner using a regret threshold that is acceptable for the DM.

3.1 Query Selection Heuristic

At any step of the elicitation procedure, we are given a (possibly empty) set $\mathcal{L}_P$ of pairs $(a, b) \in \mathcal{A} \times \mathcal{A}$ such that a is preferred to b

by the DM, filled iteratively during the previous elicitation steps. We are also given a (possibly empty) set $\mathcal{L}_I$ of pairs $(a,b) \in \mathcal{A} \times \mathcal{A}$ such that a is known to be indifferent to b through the collected preference statements of the DM. Let W be the set of weighting vectors w which are compatible with the available preference statements, i.e.

- $w \in [0,1]^m$
- $\sum_{j=1}^{m} w_j = 1$
- $a \succ b$ for all $(a,b) \in \mathcal{L}_P$
- $a \sim b$ for all $(a,b) \in \mathcal{L}_I$

The problem now consists in identifying a pair of alternatives $(a,b) \in \mathcal{A} \times \mathcal{A}$ that enables to reduce the parameter imprecision in an efficient way when presented to the DM. Note that some pairs (a,b) will be more informative than others. For instance, asking the DM to compare two alternatives a,b such that $a \succ b$ for all admissible weights $w \in W$ will provide no value as no weighting vectors will be eliminated after collecting the DM's answer. On the other hand, if the statements $a \succ b$ and $b \succ a$ are strongly supported by two different admissible weighting vectors, then (a,b) will possibly constitute a good preference query. Note that an answer of type "a is indifferent to b" is generally much more informative than a strict preference as it amounts to imposing equality constraints of type $f_w(a,p^h) = f_w(b,p^h)$, with $h \in \{1,\ldots,k\}$.

In this work, we propose to use a regret-based approach that consists in evaluating the relevance of a query by considering the worst-case loss (regret) induced when deciding whether a is ranked before b or not given the current parameter imprecision. The worst-case regret of ranking a before b is given by the strongest support of the opposing assertion $b \succ a$ over all the admissible weighting vectors. More precisely, we use the following definition of regrets:

Definition 1. *The profile-based Pairwise Max Regret of $a \in \mathcal{A}$ outranking $b \in \mathcal{A}$ according to profile $p^{\sigma(h)}$, denoted by $PMR^h(a,b,W)$, is defined by:*

$$PMR^h(a,b,W) := \max_{w \in W'} \{f_w(b,p^{\sigma(h)}) - f_w(a,p^{\sigma(h)})\}$$

where $W' := \{w \in W : \forall l < h,\ a \sim_{p^{\sigma(l)}} b\}$.

By definition, the profile-based pairwise max regret $PMR^h(a,b,W)$ is the worst-case loss induced by a outranks b when profile $p^{\sigma(h)}$ is the discriminating profile, i.e. the one leading to a preference of a over b. Then, we define the worst-case loss of a outranks b (considering all possible discriminating profiles) as follows:

Definition 2. *The Pairwise Max Regret $PMR(a,b,W)$ of $a \in \mathcal{A}$ outranks $b \in \mathcal{A}$ is defined by:*

$$PMR(a,b,W) := \max_{h \in \{1,\ldots,k\}} PMR^h(a,b,W)$$

When $PMR(a,b,W) \leq 0$, we have $f_w(b,p^{\sigma(h)}) \leq f_w(a,p^{\sigma(h)})$ for all possible discriminating profiles $p^{\sigma(h)} \in \mathcal{P}$ and all admissible weighting vectors $w \in W$. In that case, we know that a is at least as good as b for all admissible weighting vectors $w \in W$, and therefore asking the DM to compare these two alternatives is not informative. If instead we have $PMR(a,b,W) > 0$ and $PMR(b,a,W) > 0$, then we need more preference information in order to decide whether a outranks b or not. If no more question is allowed, one may want to use the following rule in order to minimize the worst-case loss: a outranks b if $PMR(a,b,W) < PMR(b,a,W)$. Therefore we define the worst-case loss of a pair (a,b) as follows:

Definition 3. *The Min Pairwise Max Regret $MPMR(\{a,b\},W)$ of a pair $(a,b) \in \mathcal{A} \times \mathcal{A}$ is defined by:*

$$MPMR(\{a,b\},W) := \min\left\{PMR(a,b,W), PMR(b,a,W)\right\}$$

Note that $MPMR(\{a,b\},W) \leq 0$ means that, as long as the DM's judgments conform to an SRMP model, the ranking between a and b is known and no query pertaining to them is needed. Otherwise, the larger this value, the larger the imprecision linked to their ranking. Asking the DM to compare a,b will lead to $MPMR(\{a,b\},W)$ dropping below 0 upon updating W using this information. For this reason, we propose the following query selection heuristic:

The Regret-Based Query Selection Strategy: At each iteration step of the elicitation procedure, select the pair of alternatives $(a,b) \in \mathcal{A} \times \mathcal{A}$ with the maximum value of $MPMR(\{a,b\},W)$.

Ideally, we would like to ask queries until $MPMR(\{a,b\},W) \leq 0$ for all $a,b \in \mathcal{A}$, which corresponds to the identification of the complete ordering of the alternatives according to the DM's preferences. To save preference queries, we could alternatively ask preference queries until $MPMR(\{a,b\},W) \leq \delta$ for all $a,b \in \mathcal{A}$, where $\delta \geq 0$ is a given positive threshold, and then use a learning algorithm to generate an SRMP model using the available preference data. Another alternative is to generate an SRMP model at every iteration step, and stop whenever the DM is satisfied with it.

3.2 Regret-optimization

In the procedure presented in the previous section, we have to compute $PMR^h(a,b,W)$ for all ordered pairs $(a,b) \in \mathcal{A} \times \mathcal{A}$ at each iteration step in order to determine the next preference query. To compute these values, we use an exact approach using a Mixed-Integer Linear program (MIP). The formulation of the MIP is given in Table 3, and its parameters and variables are respectively given in Tables 1 and 2.

$\mathcal{A}$	the set of alternatives
M	the set of criteria (m in total)
(a,b)	the current pair of alternatives
h	the current profile index
k	the number of reference profiles
σ	a permutation of $\{1,\ldots,k\}$
$\delta_j^{x\ell}$	1 if alternative x outranks profile $p^{\sigma(\ell)}$ on criterion j and 0 otherwise
$\mathcal{L}_P$	a set of pairs $(x,y) \in \mathcal{A} \times \mathcal{A}$ where x is preferred to y by the DM
$\mathcal{L}_I$	a set of pairs $(x,y) \in \mathcal{A} \times \mathcal{A}$ where x and y are considered as indifferent by the DM
γ	a small constant used to model strict inequalities

Table 1. Parameters of the MIP

w_j	*continuous*	:	the criteria weights of the SRMP model, $\forall j \in M$
s_ℓ^{xy}	*binary*	:	1 if alternative x is preferred to alternative y w.r.t. reference profile $p^{\sigma(\ell)}$ and 0 if x is indifferent to y w.r.t. reference profile $p^{\sigma(\ell)}$ $\forall(x,y) \in \mathcal{L}_P, \forall \ell \in \{1,\ldots,k\}$

Table 2. Variables of the MIP

$$
\begin{array}{lll}
\max & \sum_{j=1}^{m} w_j \delta_j^{bh} - \sum_{j=1}^{m} w_j \delta_j^{ah} & \\
\text{s.t.} & & \\
w_j \geqslant \gamma & \forall j \in M & \\
\sum_{j=1}^{m} w_j = 1 & & (i) \\
\sum_{j=1}^{m} w_j \delta_j^{b\ell} = \sum_{j=1}^{m} w_j \delta_j^{a\ell} & \forall \ell \in \{1, \dots, h-1\} & (ii) \\
\sum_{j=1}^{m} w_j \delta_j^{x\ell} = \sum_{j=1}^{m} w_j \delta_j^{y\ell} & \forall (x,y) \in \mathcal{L}_I, \forall \ell \in \{1, \dots, k\} & (iii) \\
\sum_{j=1}^{m} w_j \delta_j^{x\ell} \geqslant \sum_{j=1}^{m} w_j \delta_j^{y\ell} - s_\ell^{xy} - s_{\ell-1}^{xy} & \forall (x,y) \in \mathcal{L}_P, \forall \ell \in \{1, \dots, k\} & \\
\sum_{j=1}^{m} w_j \delta_j^{x\ell} \leqslant \sum_{j=1}^{m} w_j \delta_j^{y\ell} + s_\ell^{xy} + s_{\ell-1}^{xy} & \forall (x,y) \in \mathcal{L}_P, \forall \ell \in \{1, \dots, k\} & \\
\sum_{j=1}^{m} w_j \delta_j^{x\ell} \geqslant \sum_{j=1}^{m} w_j \delta_j^{y\ell} + \gamma - (1 - s_\ell^{xy} + s_{\ell-1}^{xy}) \times (1+\gamma) & \forall (x,y) \in \mathcal{L}_P, \forall \ell \in \{1, \dots, k\} & (iv) \\
s_0^{xy} = 0 & \forall (x,y) \in \mathcal{L}_P & \\
s_k^{xy} = 1 & \forall (x,y) \in \mathcal{L}_P &
\end{array}
$$

Table 3. MIP to compute the $PMR(a, b, p^{\sigma(h)}, W)$

In Table 3, the objective function represents the PMR of Definition (1), which has to be maximized. Constraints (i) specify that all the weights have to be strictly positive and sum up to 1, while constraints (ii) model an indifference between alternatives a and b according to the first $h-1$ profiles. Constraints (iii) (resp. (iv)) are used to represent indifference (resp. preference) judgments of the DM w.r.t. the pairs of alternatives (x, y) which were queried during the previous iterations of the incremental inference process. They restrict the possible values of the weighting vector w to subsets that are compatible with the information previously given by the DM.

4 Empirical validation

In this section, we compare the performance of our query selection heuristic (denoted by $\mathcal{H}_{mmr}$) to that of *Max Profiles* [14] (denoted by $\mathcal{H}_{mp}$) whose aim is to select a pair (a, b) that requires the highest number of profiles to be discriminated using the current SRMP model. As a baseline for comparison, we also consider the random selection heuristic (denoted by $\mathcal{H}_{rnd}$) which simply consists in selecting a pair $(a, b) \in \mathcal{A} \times \mathcal{A}$ at random at each iteration step. In our experiments, the DM is replaced by a randomly generated SRMP model (denoted by M_{DM}), which is used to compare pairs of alternatives at every step of the incremental elicitation procedures.

4.1 Design of experiments

To evaluate the performances of the three considered heuristics, we use two different sets of alternatives:

- a *training* dataset $\mathcal{D}$ of 100 pairs of alternatives, which is used as input for the three different heuristics,
- and a *test* dataset $\mathcal{D}_{test}$ composed of 5000 alternatives, which is used to evaluate the performances of the heuristics.

The performance vectors attached to alternatives are randomly drawn as integers within the interval $[\![1, 10000]\!]$ using a uniform distribution. Each heuristic is evaluated as follows: at every iteration step, the heuristic selects a pair of alternatives from $\mathcal{D}$ which is then compared using the model corresponding to the DM, i.e. M_{DM}. Using this information and that from the previous iterations, we generate an SRMP model (denoted by M) using a mixed-integer linear programming approach. Two rankings of the alternatives in $\mathcal{D}_{test}$ are then computed using both M_{DM} and M models and the *Kendall*'s rank correlation (τ) is used to compare them. This measure is used as a similarity indicator for the two rankings, and varies between -1 and 1 [13]. If both rankings are identical then $\tau = 1$, while if they are completely reversed then $\tau = -1$. The incremental elicitation process stops when all the pairs of $\mathcal{D}$ have been used, i.e. in our case after 100 iteration steps. We repeat this process 100 times (using different sets of alternatives and generated SRMP models) and report the averaged results in Section 4.1.

Implementation details. The calculations are performed on multiple servers configured with 20 CPUs (which allow reaching a parallelism of 20 when using CPLEX v12.6 solver on the proposed MIP approach) and 30 GB of RAM each. To generate a compatible SRMP model M at every step of the incremental elicitation procedures, we use an exact approach consisting in solving a MIP. Its formulation is identical to the one used to compute PMR^h values (see Table 3), except that the objective function is removed as well as constraints (ii) and their corresponding δ_j^{al} parameter values. These elements correspond to a pair of alternatives (a, b) for which the PMR^h needs to be computed, however they are not needed for the construction of an SRMP model compatible with the results fo the queries from past and current iterations.

4.2 Results

Figure 1 illustrates the mean value for the Kendall's tau measure for each iteration step of the incremental procedure, when problems with 2 profiles and 7 criteria (denoted by *(2P 7C)*) are considered, and using the three different query selection heuristics, i.e. $\mathcal{H}_{rnd}$, $\mathcal{H}_{mp}$ and $\mathcal{H}_{mmr}$.

The results show an expected trend in which the accuracy of the inferred models (given by the *Kendall's tau* values) increases with the number of pairs of alternatives being used to infer them (given by the iteration number) and tends towards 1. This is natural, as increasing the amount of information used to infer a model leads to a more accurate model. We notice that heuristic $\mathcal{H}_{mmr}$ leads to more accurate SRMP models faster than the other considered heuristics. All heuristics start rather poorly during the first iterations since fewer queries naturally lead to poor models. Towards the last iterations of the process, they also all reach mostly the same accuracy, as the entire dataset of 100 pairs of alternatives will be used to infer the models during the last iteration, regardless of the selection heuristic.

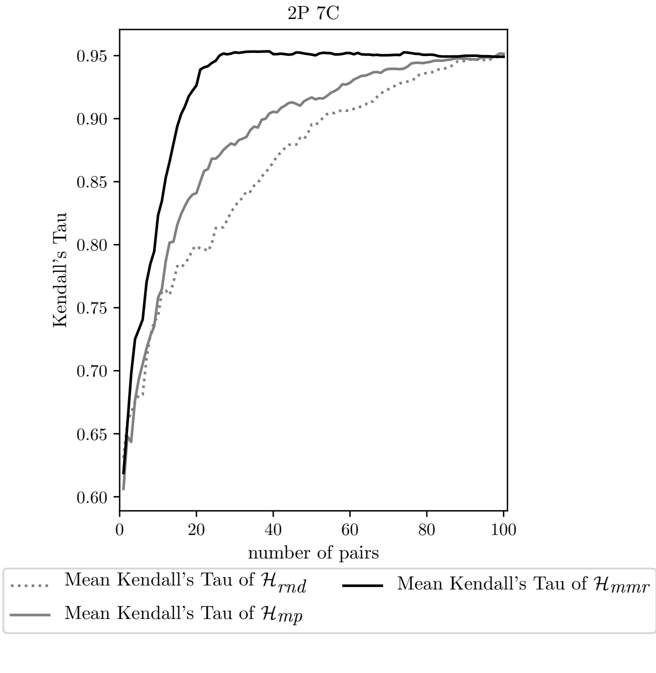

Figure 1. Mean kendall's tau for 2P 7C using the different heuristics

Figure 2 depicts the mean values of the Kendall's tau for different problem sizes. More precisely, we vary the number of profiles (k) from 2 to 3 and the number of criteria (m) among 3, 5 and 7. The corresponding problems are denoted by (kP mC) with $k \in \{2, 3\}$ and $m \in \{3, 5, 7\}$.

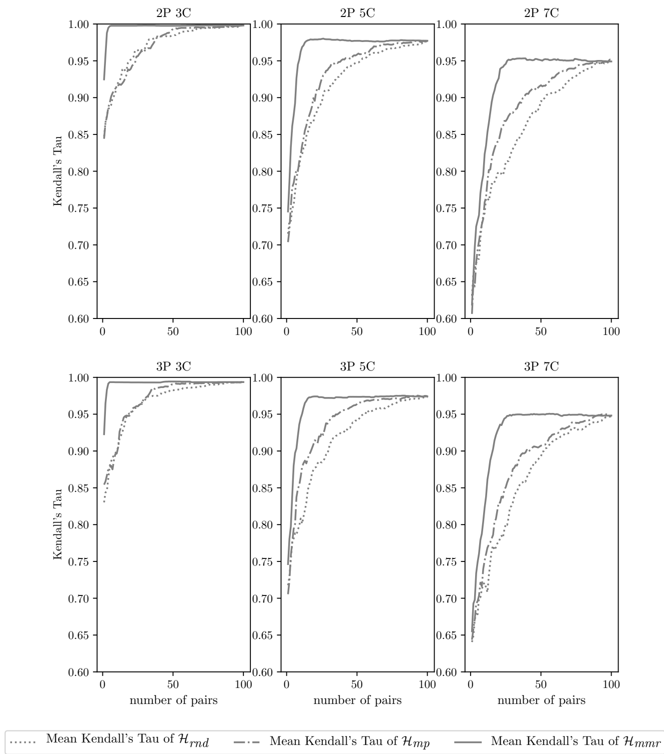

Figure 2. Mean Kendall's tau for problems of different sizes using the three heuristics

The results show the same upward trend in accuracy when increasing the number of queries as well as the fact that the proposed heuristic is significantly better then the two reference ones. We also observe that when we have more criteria, the top accuracy we can reach using the 100 pairs of alternatives decreases, potentially indicating the this set is not large or representative enough for reaching a higher accuracy. The same observation is true to a smaller degree when increasing the number of profiles.

The computation time for selecting a query is also important during an incremental elicitation process, therefore we continue by analyzing it. Figure 3 depicts the mean time for the selection of a pair at each iteration step (in seconds) for the different selection heuristics considered in our tests, for problems containing 2 profiles and 7 criteria (denoted by *(2P 7C)*). The mean time of $\mathcal{H}_{rnd}$ (dotted line) is negligible and constant across all iterations. For the $\mathcal{H}_{mp}$ heuristic, the mean time (dashed line) increases slightly with the number of iterations, however this increase cannot be perceived when looking at Figure 3. The increase, however, is due to the execution of an increasingly more complex MIP at each step, which needs to constrain more and more the set of feasible SRMP model parameters with each passing iteration. The mean time for the $\mathcal{H}_{mmr}$ heuristic (straight line) is significantly larger than that of the other heuristics. This can be explained by the fact that $\mathcal{H}_{mmr}$ executes multiple MIPs (Table 3) for each remaining pair of alternatives in the selection dataset. These MIPs also need to constrain the set of feasible SRMP model parameters as with the case of $\mathcal{H}_{mp}$, however significantly more executions of these programs need to be made within $\mathcal{H}_{mmr}$. We observe that during the initial iterations the execution time of $\mathcal{H}_{mmr}$ increases. As the number of iterations increases, fewer MIP executions are required (since fewer pairs of alternatives need to be considered), however their complexity increases (since more variables and constraints are added to the MIPs based on past queries). For this reason, the execution time starts low, increases up to a point where many pairs of alternatives still need to be considered and a significant number of past queries make the corresponding MIPs reasonably difficult to solve, while finally decreasing as fewer and fewer pairs of alternatives are left.

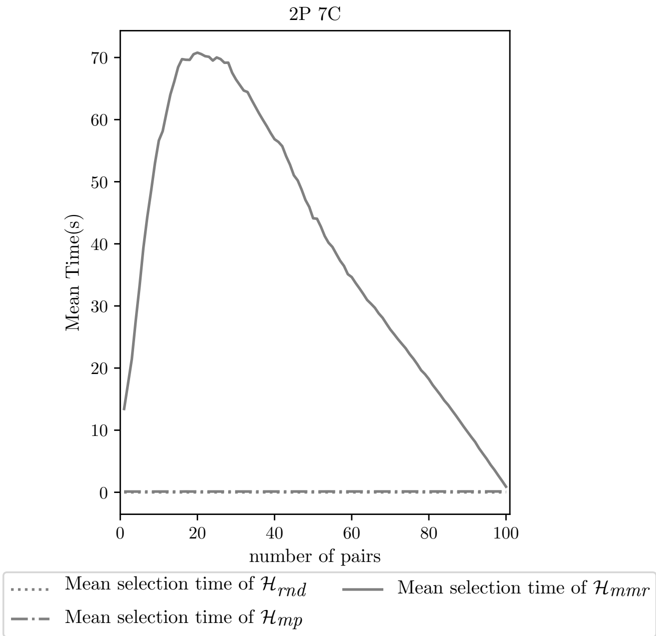

Figure 3. Mean selection time for 2P 7C

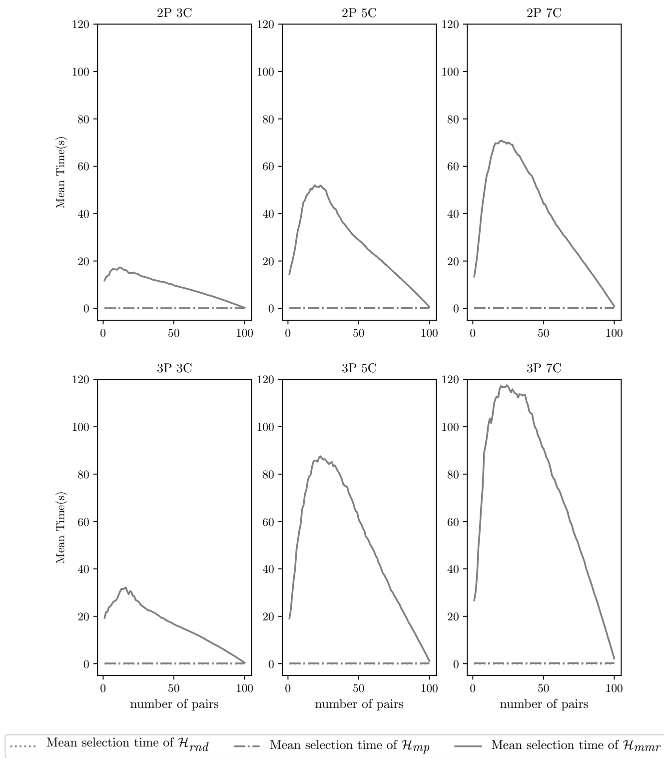

Figure 4. Mean selection time for the different problem size

Figure 3 depicts the mean time for the selection of a pair at each iteration in seconds for the different selection heuristics $\mathcal{H}_{\text{rnd}}$, $\mathcal{H}_{\text{mp}}$ and $\mathcal{H}_{\text{mmr}}$ for different problem sizes. We observe first that the different curves have the same general behavior independently of the problem sizes. We also observe that obviously the more the problem is complex in terms of number of criteria and of number of profiles, the more the computation time increases.

5 Discussion and conclusion

In this paper, we propose a new query selection heuristic, based on the minimax regret strategy, for the incremental elicitation of a ranking model using reference points (SRMP) and show, using artificial data, that it significantly outperforms existing approaches. Our aim is to be able, using this strategy, to reduce the number of queries necessary for converging to a SRMP model that accurately represents the perspective of the DM, therefore indirectly helping in easing this process.

Nevertheless, the computational effort for finding the query that minimizes the regret, as defined in this paper, can become significant, therefore we limit the learning process to only inferring the criteria weights of the SRMP model, while assuming that for the reference profiles and their lexicographic order, a direct elicitation approach has been used. Such an approach is plausible, as the profiles define independent ordinal performance intervals that should have a clear significance for the DM and which can be directly expressed by him/he more easily.

The experimental results using the proposed approach show that it outperforms previous approaches in terms of model convergence. In order to reach a 95% accuracy, it requires approximately 27 queries for problems containing 7 criteria and 2 reference profiles and around 30 queries for problems containing 7 criteria and 3 reference profiles. The number of required queries when considering smaller problems is also reduced while at the same time being lower than when using other query selection strategies. The computational time for finding a query at each iteration, however, can grow up to around one minute, however this can be mitigated, for example, by preemptively launching in parallel the calculations for finding the next query while the DM considers the current one. These calculations would need to consider all three possible outcomes for the current query (a preference in favor of any of the two alternatives, or an indifference). As soon as the DM expresses his/her preference on the current pair of alternatives, we can stop the calculations for the two no longer relevant outcomes.

All of these conclusions, however, depend on the number of pairs of alternatives to be considered, which we currently fixed to 100. Having more or fewer pairs of alternatives will naturally impact the required computational time at each iteration, therefore we will consider the topic of selecting or generating this set in order to reduce its size as much as possible in a future contribution.

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